

Relativistic Soliton Mechanics

The derivation of the Schrödinger equation, the Planck relation, the de Broglie relation, the Klein–Gordon equation and the London equation, with quantum mechanics emerging in the form of the unique low-energy limit of nonlinear electromagnetism under Lorentz invariance.

[DRAFT]

Korobochka*

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Abstract

Through a process of elimination, the sole physically and logically permissible ontology of particles emerges: vortex-type solitons emerging from the nonlinearity of the medium near the theoretically suggested Schwinger limit. The treatment of electrons not as point particles but as field disturbances naturally yields effective-mass kinematics, connecting to the wave dynamics of solitons and recovering the Planck and de Broglie relations. These relations emerge through derivation rather than postulation; χ (an electron-specific placeholder for $\hbar$) becomes a constant ratio via Lorentz invariance alone. An equation of motion for the soliton then follows, obeying both Lorentz and gauge invariance. The formulation of an action along the classical path of the soliton, combined with the most general second-order Lagrangian, yields—via physical constraints, electromagnetic gauge invariance, and internal consistency—the London and Klein–Gordon equations. The nonrelativistic limit recovers the Schrödinger equation, without the introduction of quantum postulates. Foundational experiments receive analysis within an ontic framework, concluding with a fresh treatment of Compton scattering. The final derivation shows the rest frequency of the electron proves measurable independently of $\hbar$.

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*Gensokyo @cirnosad

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1 Introduction

Standard presentations of quantum mechanics arrive laden with abstract postulates: the Schrödinger equation declared axiomatic, self-adjoint operators labeled “observables,” states identified with rays in complex Hilbert space, the Born rule for measurement outcomes, and collapse rules hastily appended for completion [1, 2]. For all their empirical success, however, such frameworks sever the physical picture from its natural roots in continuum mechanics and electromagnetism. From the severance, contradictory¹ mechanisms proliferate without ontological origin. In the end, formalism replaces understanding.

For over a century, a specific puzzle captivated de Broglie, Dirac, Lord Kelvin, Einstein, Compton, and J. J. Thomson. The present work takes up the challenge: *Assuming only Lorentz-covariant classical fields, local conservation laws, and nonlinear field dynamics, to what extent does the quantum mechanical formalism follow as a linear, low-energy approximation?*

The answer: remarkably far. The de Broglie and Planck relations, the Schrödinger equation, and the Klein–Gordon equation all emerge naturally from classical field theory once Lorentz invariance and nonlinearity enter the picture. Neither ansatz nor guesswork played any role in the derivation. Deduction leads to a single ontological foundation: the electron is a stable, localized configuration of the electromagnetic field — a soliton.

The aim here lies not in ‘proving’ electrons are solitons. The demonstration proceeds differently: solitons represent the only viable ontological candidate for experimentally observed phenomena. References to other media — elastic and superfluid continua — where nonlinearity generates solitons will support the argument. From the foregoing foundation, everything follows as corollary rather than postulate. The wavefunction ψ is constructed from scratch as a coarse-grained complex order parameter, encoding presence and internal phase of the soliton. Recognition of the soliton’s nonlocality, together with the elastic distortion of its extended structure in spacetime as demanded by Lorentz invariance, leads directly to the Schrödinger equation and the Klein–Gordon equation. Specifically, the treatment of the electron with an effective U(1) symmetry (for simplicity) yields the equations via elimination of incompatible constituent Lagrangian monomials arising from an undetermined non-linearity.

As an equivalent density and current emerge from a field-coupled Lagrangian formulation, the Born rule follows as a corollary via the ontic charge density—not via postulate.² The latter assertion follows the path of Bohmian mechanics—repeating the derivation here would be redundant, so any standard reference suffices.

None of the foregoing required energy quantization. Everything derived results from boundary conditions on solitons, removing any necessity for the label ‘quantum.’ What remains is a classical low-energy approximation emerging from nonlinear field dynamics.

¹The Schrödinger equation prescribes deterministic, linear, reversible and continuous time evolution; collapse demands probabilistic, nonlinear, irreversible and discontinuous intervention. No criterion intrinsic to the formalism demarcates the transition.

²The Heisenberg uncertainty principle receives no comment here; the result has little connection to mechanics proper, arising directly from the Fourier transformation necessary for standing wave solutions. The Poisson bracket derives from the Schrödinger equation; solutions prove orthogonal and form a Hilbert space. Nothing further need be said on the matter.

One caveat: spin lies outside the present scope, though $SU(2)$ admits parallel treatment via quaternions or 2-spinors, with phase accumulation orthogonal to spin orientation.³ Finkelstein–Rubinstein derived spin statistics for soliton-like solutions, yet this also lies beyond this paper’s remit — the essential ingredient already exist. Solitons pass through each other unchanged, gaining only a phase shift; from this property, an open path toward Fermi–Dirac statistics on ontological grounds is available.

1.1 Outline of the Paper

The paper opens with background on the present state of ontology in physics. A survey of alternate models of matter follows, establishing solitons as the sole consistent realist choice. The space of viable soliton models within a nonlinear Lorentz-invariant field then narrows; the Planck and de Broglie relations follow directly. Next, the hypothetical topology of a soliton with $U(1)$ symmetry enters the electromagnetic Lagrangian as a matter field. From this, the Klein–Gordon equation emerges as the unique non-dissipative second-order PDE linear in both electromagnetic and wave domains. Taking the matter-wave frequency as carrier and passing to the non-relativistic limit recovers the Schrödinger equation. Application to ‘foundational’ experiments demonstrates the explanatory power of the ontology.

A note before starting: the paper reads best in a single pass. The formulae will look familiar to any physicist; the derivation path will not — approaching the argument *tabula rasa* helps. Strong implications follow for distinguishing genuine postulates from what was corollary all along.

2 Background

Since antiquity, natural philosophers have pondered the composition of matter. Aristotle categorized the elements as earth, fire, water, and air — yet added a fifth, quintessence, suggested by the seemingly unchanging heavens. The concept anticipates the “aether” of later centuries. Maxwell adopted the term in 1861, positing a *luminiferous aether* carrying electromagnetic waves.

Decades later, Michelson and Morley conducted an experiment that greatly constrained the nature of any such medium: no motion relative to it could be detected. Unknown to most, Voigt had anticipated the null result months earlier, preparing what are now called the Lorentz transformations. Poincaré, Lorentz, Einstein, and Minkowski later formalized the same physics. Despite this equivalence, two distinct approaches emerged: Poincaré and Lorentz analyzed the wave equation directly, while Minkowski and Einstein imposed kinematic restrictions on a limiting speed. The term “aether” fell from favor soon after.

In 1897, the electron emerged from cathode ray experiments at the Cavendish Laboratory. J. J. Thomson, a proponent of the vortex atom hypothesis, later abandoned the idea for the ‘plum pudding’ model of the atom. Rutherford’s scattering experiments then shattered this picture: operating at far higher energies, they revealed a compact nucleus. Rutherford subsequently discovered the proton and, in the same year, hypothesized the neutron.

³Solitons based on Hopf fibrations achieve the necessary geometry, though with implications for stability.

Concurrent with these developments, the blackbody radiation puzzle prompted two successful approaches: first Wien’s displacement law, then Planck’s distribution law — the latter introducing a curious new constant. Both embedded the same core idea: the energy of modes scales linearly with frequency. The ansatz of Planck:

$$E = hf = \hbar\omega \tag{1}$$

Here h denotes the Planck constant, measured experimentally.

The photoelectric effect revealed something related: ejection of electrons from a surface depended on a threshold wavelength, not intensity.⁴ Einstein proposed that light consists of discrete energy packets with a single frequency f . An electron escapes only if the energy of the light hf exceeds the work function; the surplus becomes kinetic energy. The idea remained unpopular; Millikan’s experiment vindicated the prediction, though Millikan himself rejected Einstein’s reasoning while accepting the result.

The greatest mystery of Fraunhofer’s era was the appearance of dark lines in the solar spectrum. The phenomenon reversed in emission: heated gases produced discrete bright lines — a tool for identifying chemical elements. Johann Balmer, a Swiss schoolteacher, discovered a numerical pattern in the hydrogen lines; Johannes Rydberg later generalized the formula. In 1913, Bohr became the first to provide a physical explanation: his atomic model rendered the plum pudding picture untenable. Bohr’s insight was elegant — a standing wave condition requiring single-valued phase around any closed orbit. The observation proved so simple that Ehrenfest wrote to a collaborator: *“Bohr’s paper leads me to despair: if Balmer’s formula can be obtained in this way, I must throw all physics into the garbage heap (and also myself ...).”*

Bohr had identified the discrete states available to electrons within atoms. Mysteries persisted. Most pressing: why do electrons not spiral into the nucleus? An orbiting charge undergoes acceleration; acceleration produces radiation; radiation drains energy. Eventually every electron should collapse into the nucleus. The Bohr model stood for a decade nonetheless, though it failed to explain the Zeeman effect. Bohr and Sommerfeld attempted repairs, generalizing to elliptical orbits, but fundamental inconsistencies bred paradoxes. Working with Born, Heisenberg responded by formulating a completely different framework in 1925 (matrix mechanics) which rejected any picture of unobservable electron orbits, retaining only spectral lines and their intensities. Born applied the framework, together with the SO(4) symmetry of the hydrogen atom (encoding the conserved Runge-Lenz vector), to derive the full hydrogen spectrum. Matrix mechanics proved unwieldy; Pauli declined to compute transition intensities with the method.

Not everyone embraced the abstract framework. Two years prior to Heisenberg’s paper, Compton had performed a scattering experiment at energies exceeding all predecessors. He recognized that x-rays not only ejected electrons but transferred momentum to them, emerging with proportionally longer wavelength.

De Broglie analyzed the experiment and concluded the electron was a ‘matter-wave’ possessing both frequency and wavelength:

$$\lambda = \frac{h}{p} \tag{2}$$

Schrödinger seized on the idea. Peter Debye remarked that if the electron was a wave, it needed a wave equation. One week later, Schrödinger reported exactly such an

⁴Lenard had already established the intensity relationship.

equation, which later assumed the time-dependent form:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x},t) + V(\mathbf{x})\psi(\mathbf{x},t) = i\hbar\frac{\partial\psi}{\partial t} \quad (3)$$

The partial differential equation (hereafter PDE) was not Lorentz-invariant and could not account for spin, discovered by Stern and Gerlach years earlier. Fock, Klein, and Gordon addressed the relativistic deficiency; Dirac's equation yielded spin. Nevertheless, his formula reproduced the spectra of matrix mechanics but carried an ontological suggestion: a wave accompanying matter. Schrödinger hoped (somewhat naively) that the time-dependent form would reveal absorption and emission dynamics directly. What ψ described remained unclear. Born resolved the ambiguity, much to Schrödinger's dissatisfaction, through the probabilistic interpretation of $|\psi|^2$.

The pieces seemed to align until the 1927 Solvay conference, where Heisenberg, Bohr, and their allies effectively expelled ontology from physics through the Copenhagen interpretation. This coup caused the present interpretive morass of disconnected postulates, mutual contradictions, no account of what is being calculated. The only merit left is that predictions of the physically incomprehensible framework match experiment. For this reason, the pedagogy of quantum mechanics actively discourages inquiry into foundations — such inquiries would produce great doubts. At the center of the orthodoxy is the axiomatization of the Schrödinger equation which is said to be impossible to derive. Representative quotations follow:

“Where did we get that [equation] from? Nowhere. It is not possible to derive it from anything you know. It came out of the mind of Schrödinger, invented in his struggle to find an understanding of the experimental observations of the real world.” — Feynman (1965) [3]

“Schrödinger's equation cannot be derived, just as Maxwell's equations [cannot]. It is a wonderful result of a postulate and a guessing game based on experimental observations.” — Weng Cho Chew, Purdue (2019) [4]

“By an admittedly formal analogy, Schrödinger guessed that an electron in the external fields... would be described by a wavefunction.” — Weinberg (1995) [5]

“The Schrödinger equation cannot be derived from first principles; it is a first principle, which cannot be mathematically derived, just as Newton's laws of motion are not derivable.” — Garcia (1991) [6]

The present work demonstrates otherwise. In fairness, decades after these quotes were made, every claimed derivation of the Schrödinger equation has failed upon examination: each assumes what it purports to derive. **The derivation presented here is the first to avoid this circularity.** Most attempts invoke plane-wave ansätze and introduce the potential term without justification; others assume operator formalism from the outset. Bohm, in his *Quantum Theory* [7], employed Ehrenfest's theorem in reverse — rendering the derivation circular. Later, in developing Bohmian mechanics, he simply postulated the Schrödinger equation rather than deriving it from his ontology.

Derivations proceeding by restricting the phase velocity of an inhomogeneous wave equation must still postulate the Planck and de Broglie relations. Most critically, none

answer the foundational question: *What is ψ , and why should its spacetime evolution be constrained at all?* Such approaches violate the basic principle that ontology must precede formalism.

Consider what J. J. Thomson actually detected. He detected electromagnetic interaction with the electron — nothing more. For convenience, physicists use spheres to approximate extended objects (even cows); electrons have been no exception. Various radii emerged: the classical electron radius, defined as the scale at which electrostatic field energy equals rest energy; the reduced Compton wavelength, implied by scattering experiments. Eventually the radius was taken to zero, eliminating the paradoxes of rigid bodies yet introducing a singularity at the center of every particle.

Few know that Compton's original motivation was to investigate an extended electron model⁵, similar in spirit to what appears here. Before him, Lord Kelvin had rejected the treatment of matter as something bolted onto Maxwell's equations, seeking instead vortex solutions intrinsic to the field. The path of Compton, Kelvin, and Poincaré was abandoned for a century. The present work returns to it, but in reverse: determining what the electron *cannot* be, then converging on vortex-like solitons as the sole remaining possibility.

⁵Parson's magneton.

3 Particle Ontology

Proposing an ontological basis for physics beyond the macroscopic world grows increasingly difficult. Countless ways exist to interpret experimental measurements and attribute them to some physical picture. Yet physics concerns knowing and understanding the physical, rather than merely arriving at correct answers through random model fitting. Machine learning handles such tasks just fine and interpretation belongs in *that* realm. The aim here: an ontological basis for particles, one eliminating all other plausible choices and logically deducing the sole correct option.

3.1 Electron Structure by Process of Elimination

What are electrons made of? Most people would imagine a billiard ball marked with a negative sign and electric field lines emanating outward. Others add a magnetic field encircling the moving charge. Those familiar with the Stern-Gerlach experiment [8] know the electron exhibits an intrinsic magnetic moment aligning discretely along the axis of any applied homogeneous field, together with the magnetic field generated by its motion. The electron’s *spin* exhibits an anomalous magnetic moment $g_e - 2$ and indicates a spatially distributed charge structure incompatible with any rigid mass structure. Experienced researchers would then add screening charge and other electron-field interactions, likely wrapped in whichever framework they know best (typically Quantum Electrodynamics, QED). Frontier researchers may also consider the particle’s *Zitterbewegung*—a trembling motion at the Compton scale predicted by one interpretation of the Dirac equation.

For something imagined to be so *elementary*, electrons possess substantial structure. Yet almost all frameworks treat them as point-like particles lacking spatial extent. Some physicists believe the radius has been constrained to $10^{-20} m$, although the constraint answers the wrong question—it bounds rigid-body structure while the interaction field of the electron already has spatial extent at the Compton wavelength and classical electron radius. Such theory-laden ‘measurement’ and arbitrary exclusion of measured interaction receive little attention here.

Instead, the options themselves merit examination — starting with the conventional answer.

3.2 Point-like Particle

A point-like particle lacks extent; such a particle amounts to a coordinate in configuration space, $\mathbf{q}_i$. Attributes such as mass, charge, spin and so on are ascribed to the point. Yet point-particles fail even classically. Consider Gauss’s law: $\nabla \cdot \mathbf{E} = \rho_q / \epsilon_0$.⁶ A point particle possesses infinite charge density at the configuration point, meaning surface integration about an arbitrarily small radius yields an infinitely increasing field $1/r^2$. The field-energy of the electron becomes infinite. Were that true, a single electron could solve all of humanity’s energy problems!

Other classical problems exist. A point particle radiates according to the Abraham-Lorentz-Dirac radiation model, creating runaway accelerations and pre-acceleration⁷—

⁶The notation ρ_q distinguishes regular charge-weighted density from the wavefunction-normalized density encountered later.

⁷Causality need not be a logical necessity, but appearance of pre-acceleration remains peculiar.

causing an electron to move before a force is applied!

A significant tension is concealed by the relation $E = \hbar\omega$: the de Broglie frequency increases with velocity while any internal oscillation necessarily time-dilates. Reconciliation demands spatial structure and a point lacks an internal process for dilation. The paradoxical situation also requires the maintenance of two clocks. This precise tension prompted de Broglie's creation of matter-wave theory.

Under the Copenhagen interpretation, the configuration-space point becomes a probability wavefunction, a point appearing solely upon measurement with uncertainty – discarding ontology altogether.

In QED, the pathological situation escalates. Each loop correction diverges at higher energies—a direct consequence of integrating point-like interactions down to zero separation. Renormalization becomes necessary, absorbing infinities with arbitrary cutoffs. Such a divergence management procedure has neither explanatory power nor mathematical legitimacy. Numerical tricks do eventually align theory with experiment at the cost of both ontology and logical consistency. The infinities remain unresolved; they are merely parametrized away. The necessity of the procedure suggests failure of the point-particle assumption at short distances—precisely where validity needs to hold if electrons were *truly* points. The infinite energy problem that plagued so-called classical models persists in QED.

Within the framework of point particles, absolutely zero physical mechanism for spin exists. The problem proves serious: spin encodes an S^3 structure impossible to embed within 3-dimensional space without Hopf fibrations, Skyrmons, or other extended topological structures. Without spatial extent, spin reduces to an abstract label appended to the worldline of the point-particle. Labels are mechanisms in name alone. If the electron truly amounts to a point, where does the group action happen?

Finally, the eigenstates of the hydrogen atom reveal that s-subshells (the $l = 0$ modes) possess non-zero probability of being *inside* the nucleus. Even for point-like protons, the electron occupies the exact same space with the proton during the orbit. Were either particle a physical entity, neither could occupy a configuration-space point with the other. Where would their 'labels' be 'stored' even in the most obtuse picture? And said picture becomes further unrealistic when ascribing a trajectory to the electron—passage through the nucleus becomes necessary. Given Coulomb's law, such a trajectory creates a singularity within the motion itself. The effect proves measurable: the Lamb shift occurs precisely in such a scenario and the Dirac equation fails to generate the shift in the fine structure.

Point-like particles prove ontologically vacuous and mathematically pathological when embedded in physical theory. The field configurations of point particles, combined with their infinitesimal size, produce all kinds of infinities, demand both localized interaction *and* nonlocalized structure like gauges, and offer zero viable physical mechanism for spin. The physical implausibility of the point particle makes it, at best, an aid for calculation.

3.3 Rigid Extended Body

If a particle cannot be a point, perhaps a rigid body of finite extent works? While seemingly sensible, rigid bodies fare even worse than point-particles. Born demonstrated the incompatibility of rigid bodies with the principle of relativity. Rigid bodies also fail to exhibit the $SU(2)$ spin group action; assuming identical charge and mass distributions, the gyromagnetic ratio equals 1, contradicting the observed value of $g_e \approx 2$. Further,

straightforward calculation of necessary spin speed gives a value beyond the speed of light c . Though point-particles possess infinite self-energy, rigid-body particles possess *incorrect* energy.

Poincaré derived the equivalent mass for a given energy in the stress-energy tensor, obtaining $E = mc^2$. The equivalent energy of mass must match. Yet Max Abraham and many other scientists, attempting to derive mass-equivalent energy for a charged rigid body with a radius, arrived at $E/m = (4/3)c^2$. To resolve the mismatch, Poincaré delved into the microstructure of the electron and produced a key finding: the field at the surface constitutes the particle’s self-field and points outward, meaning the particle exerts an outward pressure on itself. With nothing to counteract this, the particle would explode. To resolve the imbalance, Poincaré added an ad hoc ‘Poincaré stress’ without physical origin, obtaining the correct equation in the 1905 paper. What do these stresses represent? Where do they come from? Poincaré offered zero answer. Without acceptance of such a mysterious mechanism, the $4/3$ problem remains unresolved.

Giving any charged particle finite extent in a rigid manner introduces paradoxes similar to what Poincaré faced. Imagine a rigid body electron made of constant charge density; draw a line in the middle. Finite repulsion exists between both parts even while in contact, with no counteracting electromagnetic force. The same holds for any plane drawn in the interior of the electron. A stable rigid body electron with any finite uniform ‘charge’ proves impossible and remains destined to explode.

3.4 The End of Matter

Poincaré did not stop at diagnosing the failures of rigid electron models. In his 1906 essay “La fin de la matière,” he followed the logic to its surprising conclusion [9]. Calculating charge-to-mass ratios via electromagnetic deflection, Poincaré noted that the apparent mass attributable to the electromagnetic field accounts for the *entirety* of the measured mass. He claimed the “real” mechanical mass—the intrinsic substance typically imagined for matter—amounts to zero. If mass is entirely the product of the electromagnetic field, and matter is inseparably connected with mass, then matter itself could not exist. Electrons, Poincaré declared, are not substances embedded in the field; they are “concavities in the aether”—holes, disturbances, patterns **in** the field rather than things floating within it.

A radical paradigm shift was crystallizing in Poincaré’s thought—a transition from the aether being a “fictitious fluid” (a mere mathematical convenience) to the proposition that it was the sole thing in existence. “In this system there is no actual matter,” he wrote, “there are only holes in the aether.” Developing the idea further in 1908, Poincaré declared conditions necessary for the principle of relativity to hold: positive electrons possess zero real mass, solely fictitious electromagnetic mass, and all forces originate electromagnetically [10]. His conclusion proved absolute: “There is no more matter, since the positive electrons no longer have real mass... The present principles of mechanics, founded upon the constancy of mass, require modification.”

Poincaré was far from alone in seeking an ontology where matter emerges from field structure rather than existing independently. In 1867, Lord Kelvin had proposed atoms were knotted vortices in the luminiferous aether [11]. Both Maxwell and Lord Kelvin drew inspiration from Helmholtz’s theorems on vortex stability—the reason Maxwell was investigating electromagnetism and added a wave equation aligning with his vortex model of the aether. Adjacent to Maxwell, Lord Kelvin used various types of knots in an attempt

to explain the periodic table of the chemical elements. The theory lost traction when Maxwell’s equations were found incapable of supporting stable vortices. Furthermore, Maxwell’s naively mechanical aether was eventually eliminated by the Michelson-Morley experiment.

Half a century later, Arthur Compton pursued a closely related idea. Before his famous 1923 scattering experiment, Compton investigated the “ring electron” model, writing a lengthy series of papers on the electron’s size and shape [12]. In the picture, the electron’s charge circulates at the speed of light c around a ring, naturally producing the observed magnetic moment. H. Stanley Allen [13], a research associate of J. J. Thomson, similarly argued for a ring electron (the Parson magneton or Toroidal ring model). Ironically, Compton’s own experiment would reverse the trend of interest in the ring electron in favor of abstract photon-electron interaction. Later, the Dirac equation was interpreted as evidence that a pointlike particle could possess intrinsic spin—though the derivation was purely interpretive rather than ontological. The ring model addressed the right questions: how does a particle have spatial extent, magnetic moment, and angular momentum? Yet it was also unable to achieve stability within linear Maxwell equations (much like a rigid body).

Decades later, quantum field theory (QFT) arrived at the same conclusion, give or take a few infinite series. In the view of recent practitioners of QFT, particles fail to constitute substances and instead represent quantized excitations of underlying fields. Hobson summarized, “*There are no particles in any of this, there are only field quanta—excitations in spatially extended continuous fields*” [14]. They went further: “*There is no particle. An electron is its field.*”. In QFT, the field proves primary and the particle secondary, with the formalism centered around operators creating and destroying particles. Unfortunately, QFT lacks an ontological framework because of field quantization, as explained by Willis Lamb in [15]—though Lamb may have intended something different. Interactions in QFT represent nonlinear effects; the most basic calculations like the Casimir effect⁸ result in infinite divergences, much like the Rayleigh model of blackbody radiation.

The critical field strength for vacuum nonlinearity was first identified by Fritz Sauter in 1931 and elaborated by Heisenberg and Euler in 1936, with Julian Schwinger later providing a systematic treatment in 1951. At such a field strength, the work done over the reduced Compton wavelength of the electron equals the electron’s rest energy, enabling spontaneous pair production. Near the limit, nonlinear electromagnetic forces become comparable to linear forces. Ideal conditions for formation of vortices. Surprisingly, nobody at the time thought to revisit the old assumptions that had previously killed research into soliton models of the electron, especially given the theoretical consensus that the field contains all the energy. Ironically, the physics of solitons and vortices was beginning to become more concrete around this very period of time. That route shall be explored.

3.5 Soliton

When Poincaré arrived at his ontology, the physicist lacked the mathematical framework for realizing physics from the ontology. His “holes in the aether” were static concavities—Poincaré possessed zero mechanism for stability of the holes and zero dynamics for sus-

⁸Which admits explanation through van der Waals forces without summing $1 + 2 + 3 + 4 + \dots$ and declaring it $-\frac{1}{12}$.

taining them. Poincaré stresses remained an unexplained fudge factor. The vortex atoms of Kelvin were topologically elegant yet required a naive mechanical aether lacking existence. The ring electron of Compton possessed the right geometry yet offered zero explanation for why the ring remained held together.

What all cases lacked was the concept of a *self-sustaining* field structure: a localized disturbance maintaining form not through rigidity or external binding forces but through the internal dynamics of the field itself—the definition of a **soliton**. A soliton needs zero ad hoc stresses; stability emerges from a balance of forces between **nonlinear** interaction and dispersion. Here lay the hiding place of the “Poincaré stress”: the field interacting with itself.

Even in the heyday of quantum fever, the idea of solitons in the field persisted. Tony Skyrme introduced soliton models in 1961 to describe nucleons. Skyrme acknowledged coming up with the idea while he “studied with attention to Kelvin’s ideas on vortex atoms” [16, 17]. The Skyrme model remains the main competitor to the enormous framework of Quantum Chromodynamics (QCD).

Solitons may seem exotic, yet most people have likely encountered or even used them. A sharp example: a hurricane, which constitutes a vortex soliton. Though hopefully never witnessed firsthand, a nuclear mushroom cloud constitutes a toroidal vortex, a type of soliton. If this is being transmitted on the internet across oceans, its likely that solitons traveling over optical fibres were used. The technique uses nonlinearities to “squeeze” pulses together during travel, enabling extremely long distance lossless communication. Caught in a traffic jam despite a clear road ahead? The cause was a shock wave caused by the delay between seeing what lies ahead and applying the brakes. A soliton within a human medium. Macroscopic solitons are pervasive.

Solitons are a plausible candidate. What remains is mapping all experimental observations onto the framework and eliminating other possibilities—yielding an ontic theory of both matter and the fields creating them. Upon aggregation of their field structure, solitons reproduce all trajectories and properties of a rigid body or point-like particle. A fascinating property of solitons: they pass through each other while retaining their shapes, with the side effect of a phase shift. The issue of passing through the nucleus ceases to break the ontic picture. The presence of solitons, moreover, aligns completely with findings in quantum mechanics when Lorentz invariance enters consideration—a point developed gradually within the paper.

3.6 Concluding the Process of Elimination

Any ontological theory of physics passes three necessary conditions:

1. logical and thus mathematical consistency;
2. correspondence between predicted observations and experimental outcomes;
3. interpretation-free status [18, 19].

The first two criteria prove obvious: any logically inconsistent framework predicts nothing and entails everything; infinitely many physically implausible theories exist. The last criterion proves subtle yet justifiable as follows. Consider a theory with two equally valid interpretations[20, 21]. Only one reality can exist, indicating the theory to be incomplete since the theory fails to distinguish between the possibilities it admits[18, 22].

The structural realist may object, claiming the criterion to be too strong; all that could be known amounts to structure between things [23, 24]. Yet such complaints belong to the realm of epistemology rather than ontology [25, 26].

For completeness, branes, strings and other topologies deserve mention. The problem: departing from conventional spacetime leaves so many degrees of freedom that an overabundance (10^{500} in string theory) of possible theories emerges. The theories become under-specified. Such under-specification fails to be ontic by definition, hence the discussion is sidestepped until the higher dimensional community produces testable theory. Loop quantum gravity, having yet to recover a smooth spacetime continuum and failing to answer the question proposed near the beginning of the current section, also gets ignored. Approaches where spacetime itself proves emergent are likewise set aside, as these demand resolving earlier questions about the substrate. In summary, these theories fail one or several of the outlined criteria.

Returning to the possibilities discussed: with strict criteria, the rigid body gets eliminated with ease—the approach can never match physically observed reality and creates logical inconsistencies. The point-particle also gets eliminated, failing all three conditions: the approach predicts infinite self-energy (unobserved) and creates all the mathematical and logical dilemmas discussed earlier. In summary, the objections of Poincaré from his 1906 essay have been demonstrated through full analysis of both ontologies: matter cannot take rigid-body form, nor compress down to an infinitely small point for elimination of paradoxes. What remains: two categories—stable field disturbances (**solitons**) or higher dimensional disturbances yet to be discovered. Any other option inevitably collapses into a theory disembodied from reality and a point-like form, or a theory requiring interpretation (quantum mechanics).

One hopes reality is real, and hence, what remains is the demonstration that a solitonic model of matter matches all known experimental findings. Other viable candidates have been eliminated through deduction and imposition of ontological understanding. Given the precision of modern particle physics measurements, such an exhaustive demonstration proves nontrivial — certainly well beyond what a lone physicist could complete in a lifetime. What proves possible, however, involves showing the viability of the path by reconstructing quantum mechanics up to 1927 with only the soliton, without Hilbert space machinery or postulates. Further, without postulating the mysterious energy-frequency Planck relation.

4 Constraining the Soliton Solution

Having demonstrated the ontological viability of solitons through a process of elimination, the class of solitons consistent with observation must be constrained. Of particular interest: **electrons**. What must a soliton constituting a disturbance of the field (a localized electromagnetic field) must reproduce?

Stability The soliton field configuration mustn't decay in low-energy interactions and, without high-energy interaction, must persist indefinitely.

Conservation of energy and charge At rest, the electron must never radiate energy. The charge of the electron must remain constant.

Electrodynamic interaction The electron must generate an electric field matching the Coulomb law in its 'far-field', spectra and intensities predicted by quantum mechanics, and also all observed nonlinear interactions of fields such as the Lamb shift and so-called screening charge in QED.

Kinematics In the role of envelope, the soliton must replicate observed kinematics of electrons including conservation of momentum.

Spin The electron must generate the experimentally measured spin dynamics and match the observed gyromagnetic ratio of the electron.

Creation-annihilation A plausible mechanism must exist for pair-production and annihilation of electrons and positrons: $\gamma + \gamma \rightarrow e^- + e^+$, $\gamma + N \rightarrow e^- + e^+$, $e^- + e^+ \rightarrow \gamma + \gamma$.

Gravity A model of solitons must eventually tackle gravity, even if quantum models have not. The aim of the ontological approach involves not matching but exceeding the performance of these models.

Interestingly enough, solitons in macroscopic systems exhibit conditions and behaviors similar to those observed during particle creation. Two oncoming non-collinear waves create a vortex soliton in water and air (a storm or hurricane). They also absorb or cancel each other out. Additionally, they possess the effective mass characteristic of solitons and seem to attract each other. Enough dynamics exist to cover the last two conditions.

What about spin? Solitons of the Hopfion class and Skyrmions realize the necessary $SU(2)$ spatially and serve as perfect candidates for the soliton so long as stability issues are addressed through nonlocality. Deep exploration of the topic exists in the literature [27]; suffice to state the idea remains viable and helps promote solitons over already discarded simplistic alternatives. A very recent paper by Oliver Consa [28, 29] produced a model of the toroidal electron yielding a gyromagnetic ratio of $2\sqrt{1 + \frac{\alpha}{\pi}}$, competing with those predicted by QED. The derivation may lack full justification (as the author acknowledges in section 5.6 of the paper), yet the method demonstrates the broad space of solutions — much unlike a point particle, which proves unable to conceivably 'spin' or exhibit action of the $SU(2)$ group.

The list specifies the *program*, though treatment of all such problems falls outside the paper's remit —certainly gravity and spin, whose full treatment requires a program of its

own. The present focus concerns the conditions necessary for the electron to exist and the physical consequences thereof, not the precise mechanisms or internal structure.

That leaves three conditions, the most pertinent being stability. First, the types of known categories of solitons and whether they map onto the observed phenomena.

5 Stability

Solitons maintain their field configuration or shape corresponding to their type. Since a convergent nonlinear description of electrodynamics remains unavailable, elimination of all candidates with certainty proves impossible. The challenge: the soliton gets defined by its topological structure and by the Lagrangian of the field containing it. The scope of possible Lagrangians cannot be onto solely what is known, like Born-Infeld; accordingly, focus falls instead on the topology *any* valid Lagrangian supports in the form of a stable solution.

5.1 Categories of Solitons

Soliton topology and dynamics admit categorization in numerous ways. By dynamic structure, solitons divide into numerous types, classes and categories:

- The **dark soliton**—an absence (self-sustaining twist or vacancy in the medium) within a continuously excited field. Dark solitons appear in optical fibers in the form of a traveling ‘gap’ within a continuous pulse.
- The ‘bright’ soliton counterpart to the dark soliton, which exists as a high amplitude excitation within the field.
- Three dimensional solitons of the Hopfion-like class and Skyrmions exist. Though a strong candidate, Skyrmions are sidestepped for convenience because they need a scalar (or group-valued) field, whereas electromagnetism remains fundamentally a vector field. That said, were a framework to work with a higher level representation such as energy density, Skyrmions may prove viable⁹.
- Q-balls. Ignored because their stability relies on a Noether charge, and they constitute solitons of scalar fields.
- Compactons. These have a strict zero boundary unobserved in a Coulomb field. All lower dimensional solitons such as kinks are eliminated as inapplicable.
- Peakons. which need discontinuous derivatives in the field (unobserved, the field remains smooth), are also eliminated.
- The **fundamental** ($N = 1$) or ‘bright’ soliton—stationary single phase frequency excitations of a field.
- Breather solitons also exist, containing multiple phase frequencies oscillating in size.

⁹Interestingly, Skyrmions emerge from large-N QCD as effective descriptions of hadrons [30].

Although the dark soliton *may* be eliminated (justified by the absence of an observed continuous field), in the inverted perspective of the medium itself everything described remains attributable to dark solitons. The distinction matters when considering the deeper microphysics of soliton formation, which lies beyond present scope. Commitment to bright or dark solitons hence proves unnecessary...

The process of elimination on known field disturbances leaves the Hopfion-like class of solitons, which include recent non-null solutions[31]. Other suitable topologies may exist yet remain unidentified—the Hopfion class remains the sole realistic choice that may be embedded within three dimensions. Hopfions prove unstable when governed by simple Lagrangians (see Derrick’s theorem) though nonlocality in the field dynamics bypasses the restriction. The Hopfion model is a preferred working model as Skyrmsions may fail to produce all necessary symmetries without additional structure.

What remains: tackling the final category of whether the soliton constitutes a breather or fundamental soliton. Accordingly, what follows examines the observable consequences emerging from an electron breather, and why exclusion of the possibility proves necessary. The distinction proves essential for subsequent analysis; careful treatment remains necessary since the outcome cascades into all further physics.

5.2 Fundamental vs Breather Soliton

If an electron were a breather soliton, the electron would possess multiple or at least two phase frequencies. The energy within circulates at different rates, creating patterns of destructive interference varying temporally in the near-field. Electromagnetically, even under any conceivable nonlinear conservative dynamic, these result in measurable patterns of far-field interference. Since the soliton presents a field with a charge, both frequencies ω_0 and ω_1 beat against each other, creating a signal $|\omega_1 - \omega_0|$. Such a signal has never been measured. Additionally, such coupling presents a radiative path for the electron at rest, meaning the electron gradually loses energy and decays. Such decay goes unobserved.

That electrons constitute breather solitons proves inconceivable. Electrons are therefore fundamental. Such exclusion of course leaves breather solutions viable for *other* particles. For instance, “neutrinos” are said to exhibit a mysterious ‘oscillation’ between types of neutrinos. The phenomenon may involve two different frequencies coupling very weakly to an external field. Oscillations of such kind go unobserved between electrons and other similar unstable particles like muons.

A fundamental solitons can be modeled with a rest frame phase frequency of ω_0 . Harmonics of the frequency may exist but strictly as a polarization rather than a internal flow of energy. A contrasting example: the Zitterbewegung, running at twice the phase frequency¹⁰, remains permitted so long as the Zitterbewegung is restricted to non-energy flow. In general, any breather in some higher dimension at harmonic frequencies remains permitted provided the extra harmonics fail to represent any outward flows of energy.

Let us assume the opposite and that such a secondary mode couples weakly to the far-field, producing radiation below available detection thresholds. Such objection fails on energetic grounds. Any nonzero radiative coupling, however feeble, implies continuous loss of energy and thus finite lifetime. Yet the experimental lower bound on electron stability exceeds 10^{28} years, with zero observed change in mass at the level of one part in 10^{18} [32, 33]. Stability is absolute. The secondary mode of a hypothetical breather

¹⁰According to the Dirac equation.

electron demands exact non-radiative character: perfectly decoupled from the electromagnetic far-field despite constituting a configuration of an electromagnetic field. In linear electromagnetism, any time-varying multipole moment radiates, prohibiting a secondary mode. One may speculate strong nonlinearities near the Schwinger limit permit “dark” internal modes whose oscillations remain strictly confined—yet such modes need a mechanism whereby redistribution of energy within the soliton produces zero net change to any multipole moment at infinity. Since zero such framework exists, a single phase frequency of energy constitutes the prudent deduction.

Since breather solutions naturally decay, they constitute suitable candidates for unstable particles. In the future, such dynamics may tame the unwieldy “particle zoo” classifications of mass and lifetime.

5.3 Nonlinear Plausibility

Solutions to Maxwell’s equations forming standing waves exist (Rañada Hopfions, solutions of Arrayás-Trueba etc.) although they disperse instantaneously. Thus, the structure generated during pair creation must persist through a balance of the nonlinear seemingly ‘attractive’ dynamics and dispersion. The fields must close in on themselves while simultaneously remaining together at rest. Putting experimentally derived limits on these vague notions would require testing modified Maxwell equations such as the Euler-Heisenberg equation at energy scales currently unavailable (approaching the Schwinger limit)—and likely to remain so for a very long time. In these formulations, the “vacuum” itself acquires an effective polarization:

$$\mathcal{L} = \varepsilon_0 \mathcal{F} + \frac{\alpha}{90\pi} \frac{\varepsilon_0}{E_c^2} [4\mathcal{F}^2 + 7\mathcal{G}^2] + \dots \quad (4)$$

where $\mathcal{F} = \frac{1}{2}(E^2 - c^2 B^2)$ and $\mathcal{G} = c \mathbf{E} \cdot \mathbf{B}$ are Lorentz invariants, and E_c denotes the critical Schwinger field. The Euler-Heisenberg Lagrangian represents the lowest-order QED perturbation to Maxwell’s equations, and the critical Schwinger field has an interesting relation to the electron specifically:

$$e \cdot E_c \cdot \bar{\lambda}_c = m_e c^2 \quad (5)$$

At the reduced Compton wavelength ($\bar{\lambda}_c$), the work done by the Schwinger field equals the rest energy of an electron. Such correspondence proves suggestive, though the correspondence remains experimentally untested up to the field strength and much of QED-based tests prove heavily theory-laden. Stability remains viable in the realm of known physics insomuch as it is not eliminated from the realm of possibility. Ironically, were the technology necessary for testing fields of such intensity ever built, the critical field of pair creation gets entered, interfering with or destroying the electron being measured!

5.4 Further Notes on Conservation of Vorticity

With a single frequency fundamental soliton established the question becomes: despite Lorentz invariance and conservation of energy, why does the frequency remain constant when acted upon by energetic events? In other words, what makes the frequency a **characteristic** of the particle, like mass, charge, and rest energy? Were the electron to shift the overall frequency in the rest frame of the electron, the topological charge shifts as well, and the shift results in decay because the process can be repeated continuously

until the charge vanishes. In reality, the singular event that may change the frequency of a soliton involves collision with its anti-soliton (for the electron, a positron), which possesses vorticity of opposite direction. Therefore, the same unknown nonlinear and nonlocal restorative force from which the soliton emerges must also conserve its vorticity, and hence its frequency.

Pertinent to the discussion, macroscopic solitons exist in superfluids called “quantum vortices”; a photograph of such a phenomenon together with an illustration appears in figure 1. These solitons exhibit pair-antipair creation, charge quantization, attractive and repulsive forces, and can normally be annihilated solely by an anticharge, rendering them robust topological defects¹¹.

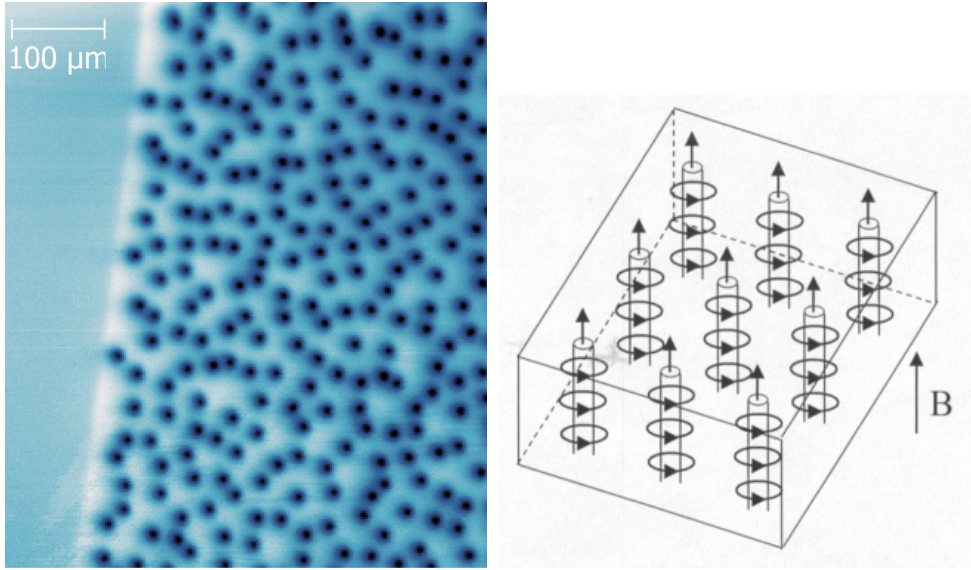


Figure 1: Left: Lattice of quantum vortices observed in a Bose-Einstein condensate. The dark spots are the cores of quantized vortices, where the density of the superfluid vanishes. Right: The vortices form a partially ordered structure of an Abrikosov-like lattice.

5.5 Experimental Verification

Now that the type of soliton and a single phase frequency has been established, a natural inquiry arises: can ω_0 be measured independently instead of being inferred indirectly? Can the frequency survive high energy events of a non-topological defect nature? If the answer proves negative to either, the frequency remains a purely theoretical aid instead of an ontic entity. Remarkably, the frequency has already been measured, though without wide recognition. A historical analysis of the experiment demonstrating said measurement appears later in the paper, together with a complete justification for the directness and independence of the measurement. That very same experiment demonstrates the robustness even with energy impacts exceeding the equivalent mass-energy of the electron.

¹¹They also exhibit Lorentz invariance, which rendered the analogy so vivid it inspired Russian theoretical physicist Grigory E. Volovik to write the provocative yet interesting book “The Universe in a Helium Droplet” [34]. In the book Volovik mentions quasiparticles obey the law $E^2 = c_\perp^2 p_\perp^2 + c_\parallel^2 p_\parallel^2$ which mirrors the Planck relation and encodes Lorentz invariance.

6 Conservation and Kinematics

6.1 Charge

How can a soliton—an electromagnetic disturbance—possess charge? The answer proves straightforward yet rests on an interesting premise. Charge is measured by the force exerted on another charge; Coulomb’s law captures precisely this principle and remains valid for the electric displacement D in Lagrangians like Euler-Heisenberg and Born-Infeld. A field configuration that, beyond a reasonable interaction radius (the reduced Compton wavelength, say) and hence beyond contributions from nonlinear effects, resembles exactly a Coulomb field for the electron charge e yields the following surface integral over D :

$$\oint_{S(\bar{\lambda}_c)} \mathbf{D} \cdot d\mathbf{S} = q = -|e| \quad (6)$$

In the far-field, the regular Maxwell equations apply:

$$\mathbf{E}(\mathbf{r}) = E(|\mathbf{r}|)\hat{\mathbf{r}}, \quad E(r) = \frac{q}{4\pi\epsilon_0 r^2} \quad (7)$$

The resulting expression must be without radiative ($1/r$) terms, otherwise the terms would fail to contribute to the charge and would produce energy loss instead. An interesting conclusion emerges from the preceding result, though somewhat obvious upon reflection. Conception of electrons in the form of rigid or point-like particles led naturally to searches for the radius at which electrons existed. Stripping successive layers of interaction from methodology and experiment yielded results indicating a vanishing radius. Under the present treatment, however, the radius proves *infinite*, spanning all of spacetime (for an infinitely old electron, at least), because the electron *constitutes* the field creating the interactions—and the field possesses no outward limit.

Only one electromagnetic field exists, infinite in expanse. The electron’s field likewise extends infinitely and remains embedded within the one electromagnetic field. The two infinite fields cannot differ—accordingly, a soliton electron *constitutes* the field, exactly in the manner Poincaré posited. The one-electron universe of John Wheeler receives a nod here, in a sense, though not with the exact temporal dynamics he had in mind.

The first obvious restriction on what the soliton must generate follows: the Coulomb field beyond some radius. A field resembling a Coulomb field creates an effective charge in exactly the manner a point-like or rigid particle would.

6.2 Mass

With charge addressed, mass remains. The non-trivial aspect of the treatment enters here. Poincaré observed that mass remains coupled with charge in all electromagnetic interactions. Inspection of the Lorentz force law reveals the reason:

$$\mathbf{F} = m \frac{\partial \mathbf{v}}{\partial t} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \Rightarrow \frac{\partial \mathbf{v}}{\partial t} = \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (8)$$

A nonlinear (and nonlocal) Lagrangian providing the particle with momentum would create mass. Such an approach would unfortunately involve selection of a particular Lagrangian and a particular soliton form, both of which have been left unspecified. The two remain closely linked, given that the solution would require a timelike momentum

product $P^\mu P_\mu = m^2 c^4 > 0$. The Poincaré stress problem returns in a convoluted form. While avoiding speculation, it must be stated that the likelihood remains that the question holds the key to unlocking the nonlinear restorative force of solitons, through a nonlocal medium rather than field interaction.

Two separate properties characterize macroscopic vortices and solitons: a charge and a mass, in the same manner as the electron soliton. The charge arises from a topological or temporal quantity such as winding number, whereas the mass emerges from nonlinearity in the medium. Foreshadowing the discussion to come: might mass result from a bilinear operation on current density? Something of the form:

$$E = (\sqrt{m}c) \cdot (\sqrt{m}c) \quad (9)$$

Battles must be chosen wisely, and endless time has been fruitlessly poured into this question. The origin of mass is therefore left out of the remit of this paper. The electron rest mass then follows from the relation $m_0 = E_0/c^2$ without further explanation.

6.3 Energy

The soliton possesses strictly finite field energy, and unlike the infinite self-energy for a point particle, the generation of a Coulomb law below an interaction radius is unnecessary. The ‘interior’ of the electron then constitutes a topological object in the nonlinear field, remaining undetermined. For any Lorentz-invariant Lagrangian, the energy flow is derived from the Umov-Poynting vector $\mathbf{S}$ and the energy density u :

$$u(\mathbf{r}, t) = \frac{1}{2}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H}), \quad \mathbf{S}(\mathbf{r}, t) = \mathbf{E} \times \mathbf{H} \quad (10)$$

Stability requires that the energy of an electron remain constant. The energy integral accordingly is applied in a time independent fashion within the rest frame of the particle:

$$E_0 = \int_{\mathbb{R}^3} u(\mathbf{r}, 0) dV \quad (11)$$

The energy density scalar u and the Umov-Poynting vector $\mathbf{S}$ play the same role in the Maxwell equations as the charge density ρ and current density $\mathbf{J}$. Expression of said current density in covariant form $J^\mu = (c\rho_q, \mathbf{J})$, together with consideration of the four-stress-energy tensor $T^{\mu\nu}$, yields the same continuity equation:

$$\partial_\mu J_q^\mu = 0 \quad \partial_\mu T^{\mu 0} = 0 \quad (12)$$

Here $T^{00} = u$ and $T^{0i} = \frac{1}{c}S^i$ (the signature $(+, -, -, -)$ applies throughout the text). The resulting continuity equation matches identically the form for charge and current density, containing no additional ‘source’:

$$\frac{\partial u}{\partial t} + \nabla \cdot \mathbf{S} = 0 \quad (13)$$

The magnetic field at rest of an electron is zero, hence the energy flow $\mathbf{S}$ should also go asymptotically to zero at some critical radius. The interior flow of $\mathbf{S}$ must be closed to avoid far-field radiation. Standard null Maxwell solitons, being radiative, fail the latter requirement. Balancing nonlinearity against dispersion presumably requires distortion of the field for a particle-type soliton.

An obvious yet decisive point emerges after this section and the one prior: the electron possesses both a rest energy E_0 and the rest phase frequency ω_0 . This result is provably decisive as shall be discovered shortly.

7 Lorentz-Invariant Wave Kinematics

The phase frequency and energy at rest are intrinsic to the soliton field, which may be interpreted as a resonance. Certainly, ever since the Millikan experiment it is known that the electron's charge is universal. The mass of the electron was likewise found to take a universal value in chemistry and nuclear physics. Experimental evidence for a frequency intrinsic to the electron has existed for over a century, dating back to the Compton scattering experiment.¹² For a fundamental electromagnetic soliton, the phase frequency constitutes not merely a mathematical artifact but an ontic property of the electromagnetic process.

Given that the energy and phase are both intrinsic to the particle at rest, one may construct the following ratio out of two Lorentz-invariant constants in the electron rest frame:

$$\chi \equiv \frac{E_0}{\omega_0} \quad (14)$$

Consider what happens to the energy E_0 when boosted to velocity v along the x axis from rest. What does v mean for the soliton, given that the soliton constitutes an electromagnetic mode? Where does the electron reside? By isotropy of field symmetry, the energy density should be uniform in all directions beyond some critical radius. Hence, the center follows from an inertial frame integral:

$$\mathbf{x} = \frac{1}{E} \int_{\mathbb{R}^3} \mathbf{x} T^{00} d^3x \quad (15)$$

A soliton possesses spatial extent and Lorentz invariance accordingly forbids its rigidity. Yet, the soliton *does* retain shape within the rest frame, with all energy flowing at light speed c internally, otherwise such rest frame deformation results in an outwards energy flow. Consider a coherent frame and a sufficiently large radius of integration to yield a velocity:

$$\mathbf{v} = \partial_t \mathbf{x} = \frac{1}{E} \int x^j \frac{\partial T^{00}}{\partial t} d^3x \quad (16)$$

The continuity equation, together with integration by parts and the condition of non-radiation in the limit $r \rightarrow \infty$, yields the result:

$$\frac{\partial T^{00}}{\partial t} = -c \partial_i T^{i0} \implies \mathbf{v} = \frac{c}{E} \int T^{0j} d^3x \quad (17)$$

The definition of a vector component of P^μ is $P^j = \frac{1}{c} \int T^{0j} d^3x$, and so the following relation emerges:

$$\mathbf{v} = \frac{c^2}{E} \mathbf{P} \quad (18)$$

Hence, the usual kinematic relation emerges. A soliton at rest accordingly possesses a well-defined 4-momentum P^μ :

$$P^\mu = \left(\frac{E_0}{c}, \mathbf{0} \right) \quad (19)$$

¹²Of interest: some have also attempted measurement of the Zitterbewegung frequency [35, 36], but the result remains somewhat disputed.

A boost of velocity v along the x -axis yields the known result:

$$P^\mu = (\gamma \frac{E_0}{c}, \gamma \frac{E_0 v}{c^2}, 0, 0) \quad (20)$$

The usual transformations of energy and momentum follow:

$$E = \gamma E_0 \quad (21)$$

$$p_{\parallel} = \gamma E_0 \frac{v}{c^2} \quad (22)$$

The procedure may seem pedantic as it arrives at an otherwise known result — but the obviousness holds only for a *point-particle*. A soliton possesses additional structure, and the structure reveals a relation most take for granted.

A soliton extended into 3 dimensions possesses wavenumber vectors $\mathbf{k}$ in all directions, with the wavenumber necessarily aligned with the flow of energy $\mathbf{S}$. The energy flow must circulate¹³, and hence for any particular axis of boost (the x axis, for instance) there must exist a point in the rest frame of the soliton with a 4-vector $k^\mu = (\frac{\omega_0}{c}, 0, k_y, k_z)$ possessing a space component orthogonal to that axis of boost. The following Lorentz-invariant phase for the direction of boost can be thereby constructed:

$$\phi = \omega_0 t' - k_y y' - k_z z' \quad (23)$$

Here t' denotes time relative to the rest frame of the soliton. The local ϕ phase is a Lorentz scalar, so a axial displacement x can be chosen within the soliton without loss of generality. Next, the soliton is boosted with velocity v in the x direction through application of the usual inverse Lorentz transformation:

$$\begin{aligned} t' &= \gamma(t - \frac{1}{c^2}vx) \\ x' &= \gamma(x - vt) \\ y' &= y, \quad z' = z \end{aligned} \quad (24)$$

$$\phi = \omega_0 \gamma(t - \frac{1}{c^2}vx) - k_y y - k_z z = \gamma \omega_0 t - \frac{\gamma \omega_0}{c^2}vx - k_y y - k_z z \quad (25)$$

In the moving frame, the space components of k^μ have gained an x component. The 4-wavenumber accordingly becomes:

$$k^\mu = (\gamma \frac{\omega_0}{c}, \frac{\gamma \omega_0}{c^2}v, k_y, k_z) \quad (26)$$

Given that the direction was arbitrary, and the aggregate momentum would otherwise integrate to zero by symmetry, the following transformations emerge:

$$\omega = \gamma \omega_0 \quad (27)$$

$$k_{\parallel} = \gamma \omega_0 \frac{v}{c^2} \quad (28)$$

Division of equation (21) by (27) yields the following relation:

$$\frac{E}{\omega} = \frac{\gamma}{\gamma} \cdot \frac{E_0}{\omega_0} = \chi \quad (29)$$

¹³As discussed earlier.

Similarly, division of (22) by (28) yields the relation:

$$\frac{p_{\parallel}}{k_{\parallel}} = \frac{\gamma v/c^2}{\gamma v/c^2} \cdot \frac{E_0}{\omega_0} = \chi \quad (30)$$

Rearrangement yields:

$$E = \chi \omega \quad p = \chi k \quad (31)$$

With the derivation complete, the Planck and de Broglie relations have been obtained without postulation. The relations normally get postulated; a significant and new result. The finding vindicates selection of the fundamental soliton type, given that a solution with multiple phase harmonics would lack a defined frequency and hence could not generate the requisite phase relation—a point aligned with experimental results in the penultimate section.

Strictly speaking, the p and k entering (31) are defined at different levels: p was obtained through an energy-weighted integral of T^{0j} over the soliton, whereas k^{μ} in the derivation above was a local 4-vector in a particular patch. Comparison requires definition of an effective wave 4-vector K^{μ} for the soliton by coarse-graining the local $k^{\mu}(x)$ field with the same energy density defining P^{μ} . The boost adds the same longitudinal component to every local k^{μ} in the rest frame, and accordingly the averaging preserves proportionality, yielding $P^{\mu} = \chi K^{\mu}$ for the coarse-grained quantities.

In fact, only the E relation is needed; a general result for any overall K^{μ} given an E and ω relation (demonstrated above) follows—for instance, see [37]. Notably, the frequency increases with particle boost, which is not only a feature of the model but also an implication of the relationship. Ordinarily, a boost slows time for the moving particle (observed in radioactive decay), yet embedding the “clock” in the form of a phase frequency causes the boost to act in an inverse manner. To reiterate a point made previously, this realization spurred de Broglie to formulate the matter-wave hypothesis.

The relations in (31) are arguably the most significant result of this paper. The Planck relation is normally claimed to be the ‘first quantization’ step, rendering otherwise classical treatments ‘semi-classical’. Anyone adopting the soliton model up to this point may therefore declare their model **fully classical** since no ‘quantum’ assumptions have been smuggled in. A point of interest: *these relations bridge waves to kinematics* and mediate physical connections between the two domains. With this complete, an ontology for observations can be constructed. The relation has been demonstrated, but the experimental determination of the value $\chi = \hbar$ nor the detailed origin of its particular value. Furthermore, the derivation applies to only one soliton species; no claims about universality of internal-frequency relations between species have been put forward.

This is even true experimentally, as electrons are used for all measurements and hence this relation likewise enters all such measurements with the same ratio mediating the bridge between kinematic and wave phenomena. Electrons reside in the detector or another part of the system being measured.¹⁴ There is one exception, Muonic hydrogen transitions and it just so happens to demonstrate numerous ‘anomalies’ within popular frameworks.

¹⁴The Planck relation is normally applied to ‘photons’, but photons prove not ontic. Good company exists for this view: Willis Lamb (winner of the Nobel prize, discoverer of the Lamb shift) wrote an entire article *anti-photon* explaining why the concept necessarily constitutes a misnomer. Objections including anti-bunching get addressed in appendix A.

To connect χ to experimentally observable phenomena, it is necessary to demonstrate that solitons exhibit the exact behavior observed in quantum mechanics. Hence, the derivation of the Klein–Gordon equation from foundational axioms via elimination over the only possible linear form, is the next step. The Schrödinger equation follows, with χ matched to $\hbar$ in the final step.

Prior to these steps, an appropriate representation for the soliton must first be defined. Solitons are not rigid, in the manner emphasized repeatedly—yet in Lorentz-invariant spacetime this produces a strange phenomenon, little discussed (probably due to the obsession with particle-based treatments of Einstein–Planck relativity and ‘worldlines’): the soliton deforms not only in space but also in **time**. As a result, the field disturbance is distributed and mixed within time in a manner hidden in explicit order, only to be revealed as an implicit order only once a significant event forces the restoring force and soliton field configuration to react coherently in the form of one unified body.

8 Wave Mechanics: Bridging Lorentz Invariance to Kinematics

8.1 Spacetime Deformation

Most treatments of relativity begin with the postulate-based treatment of Lorentz invariance by Einstein and Planck, kinematic in nature. Historically, the Lorentz invariance relationships were extracted from a class of hyperbolic second-order wave equations:

$$\square_c \phi = 0 \tag{32}$$

This PDE supports a **single** speed: c in all directions. The Lorentz group comprises precisely those linear transformations (functions of boost velocity) keeping the wave object ϕ invariant. For waves the unique transformation group of coordinate that preserve the underlying physics for a coordinate changed ϕ' is the Lorentz group. To be specific, the same separation of wave fronts up to transformation should be observed in each inertial frame chosen, and equivalently, the phase must match up to an additive global constant. Treating matter as separate to waves necessitates a logical bridge to maintain physics in all inertial frames, as forces the matter to adhere to this invariant and such a constraint constitutes the ‘postulate’ of relativity.

Pedagogically, one often represents motion by worldlines and point events. The underlying assumption that permits this simplification is the representation of particles as points. Such idealisations are inappropriate for solitons, which are a persistent excitation of the underlying field ϕ — a wave. Although a soliton is not ontologically separate from the field that supports it, isolating it as a distinguishable structure is useful for tractable modelling. Yet an unnecessary postulate may be discarded here: no requirement exists to ‘limit’ any speeds or find preferred frames. This follows from being a solution to the wave equation and the soliton being embedded as ‘matter’ in that solution — the observers included of course. The medium cannot be separated from the waves it hosts and so additional postulation such as Einstein’s is, to use his favorite word, superfluous.

The circulation of energy within a soliton travels at light speed c ¹⁵, yet since the circulation remains closed (nonlinear forces balanced against the dispersion of the regular hyperbolic wave equation), the envelope of the soliton can move at speeds up to c . One catch: relativity forbids rigid objects, so whereas the soliton remains an undeformed shape in its own rest frame, each part of the soliton experiences space-varying boosts according to the effective potential around it. Such a restriction forbids usage of the usual tools—the worldline ceases to correspond to reality, even for one lone particle. What is required is a *worldtube*, which appears schematically in figure 2.

This may be counter-intuitive: a naive reading requires the maintenance of the soliton shape under motion, lest the conserved topological charge be lost. Yet, the soliton *does* keep its shape perfectly—an observer within the rest frame of the soliton (should the soliton move as a whole, coherently) sees a matching particle. When experiencing uneven acceleration, it becomes impossible for an observer traveling at identical speed as a given point within the soliton to occupy a rest frame—yet the soliton *does* exist within the object ϕ as a coherent wavefront. The moment some violent acceleration forces the leading part of a wave to reverse direction, the soliton coheres and remains connected.

¹⁵The only speed available.

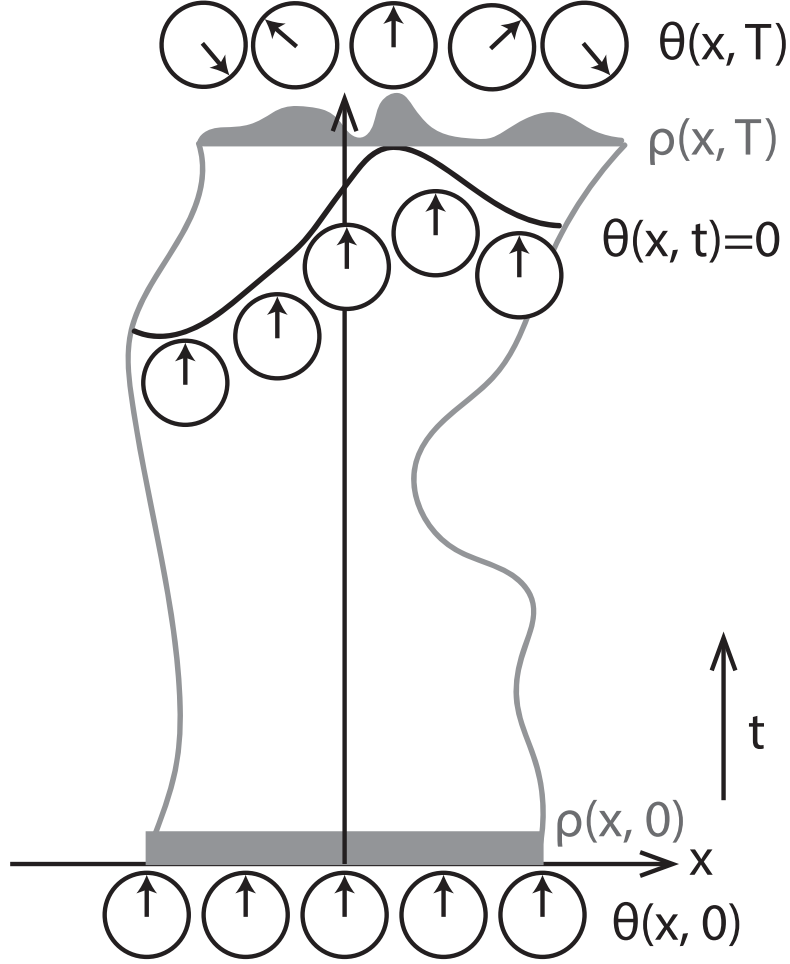


Figure 2: Worldtube of spacetime of a soliton undergoing motion with a gradient of phase or uneven acceleration. The arrows at the bottom depict synchronised ‘rest frame’ clocks where the soliton is coherent in the lab frame, $\theta(x, 0)$. The gray block represents the charge density $\rho(x, 0)$. Continuous contraction of length and dilation of time cause the phase of the soliton to decohere and the effective charge density to continuously morph in spacetime. An example wavefront is plotted centrally, where phase remains constant while x and t trace a continuous path showing the connected particle. This corresponds to the rest frame of the soliton at that instant. At $t = T$, the lab frame has a distribution over θ and ρ .

Continuity is maintained via the the same nonlinear forces keeping the wave together in its original phase-coherent rest frame.

What happens when the soliton gets spread far while traveling at too high a speed for reassembly for the time needed to remain whole?¹⁶ Under an ontological approach, such a scenario poses zero difficulty since causality goes unpostulated¹⁷; experimentally verified facts are the sole guide of analysis. The entangled state of the field and a nonlocal nonlinear restoring force may ensure the event happens as observed, just as in quantum mechanics.

The ensuing discussion has hopefully illuminated predicting the kinematics of solitons proves likely *intractable* without further foundational discoveries, ingenuity and mathematical techniques.¹⁸ The ‘violent events’ constitute the sole means by which the particle localizes in a meaningful way, so even determining the exact dynamical form remains exceedingly difficult. Yet, by carefully representing the object of the soliton, predictions about motions of the soliton, states of energy and more remain accessible.

Proceeding demands a paradigm shift: the electron soliton cannot be treated as a “point particle”. It must be approached as a steady-state fluid dynamics problem. Any valid and applicable representation must capture both the phase coherency in the rest frame and the spreads of the electron through its spacetime worldtube.

8.2 Representation

Consider the charge density of the particle at rest. Earlier, the stress-energy tensor was employed to generate an aggregated velocity from the energy density of the soliton. The same idea yields a charge density. If the total charge is q , the density of charge is:

$$\rho(\mathbf{x}, t) = \frac{T^{00}(t, \mathbf{x})}{\int T^{00}(t, \mathbf{x}) d^3x}, \quad \rho_q(\mathbf{x}, t) = q\rho(\mathbf{x}, t) \quad (33)$$

The density proves strictly positive or zero everywhere, a coarse-grained approximation since the structure inside the soliton remains unknown. The simplified model proves valid where interaction doesn’t involve lengths below the reduced Compton wavelength, or direct interaction with other solitons (such as passing through a nucleus).

At rest, the density takes on an exact form matching the term of energy density in the stress-energy tensor. The moment the soliton moves, the density gets smeared over spacetime. Each motion undertaken uniquely modifies the density function, which spans all of spacetime. The density function constitutes a partition of ϕ , excluding everything ‘external’ to the soliton. Yet the function is an incomplete description of the soliton, as it is electromagnetic in nature and oscillates at a constant frequency ω_0 . At the lowest order, the phase evolution and coarse density must be modeled.

A proper treatment involves tracking the phase at every point, and the phase possesses spacelike coherence in the rest frame (a frame that, strictly speaking, exists solely at

¹⁶The measurement collapse problem, in essence, which despite the moaning and groaning of the pedagogues, violates their precious causality, unless one redefines it as the completely arbitrary “transmission of information”. The standard line of argumentation inevitably leads to circular definitions or contradictions [38].

¹⁷One might assume causality comes as part and parcel with Lorentz invariance, although it does not. Lorentz invariance possesses zero temporal directional preference either.

¹⁸An exact understanding of the soliton helps answer some questions about nuclear interactions; a nonlinear PDE likely produces insight about chemical bonding without empirical input. Potentially, such insight may even offer explanatory power in the domain of exotic nuclear reactions.

creation of particles, or when the particle somehow gets brought to a complete stop everywhere simultaneously)¹⁹. The phase exists in a compact manifold rather than a linear space as phases 0 and 2π represent the exact same phase. The phase maps to a point in S^1 , the quotient space $[0, 2\pi]/(0 \sim 2\pi)$. Meanwhile, the polarization correspondence between $\mathbf{E}$ and $\mathbf{B}$ maps onto a direction on a unit sphere, or the S^2 manifold.

Now were the soliton to lack structure or the polarization and phase to be independent, these two could be combined by taking a product of the manifolds, $S^1 \times S^2$. This is not the case and the observed behavior necessitates a S^3 manifold, embedded in 4 dimensions in its simplest form. The Hopf fibration can map this back to S^2 and allow embedding within 3 dimensions at the cost of self-intersection. The equivalent group action is then, $SU(2)$. A full representation needs quaternions, or a two-dimensional complex vector (spinors). As derivation of the Klein–Gordon equation is the current aim, polarization gets put aside with focus being restricted to the temporal phase alone.

A representation of the phase $\theta(\mathbf{x}, t)$ must incorporate phase circulation. Let $\mathbf{x} = \mathbf{0}$ denote the center of the soliton. The phase at this point, expressed as a function of proper time τ , is given by:

$$\theta(\mathbf{x} = \mathbf{0}, \tau) = k_\mu x^\mu + \theta_0 = \omega_0 \tau - \mathbf{k} \cdot \mathbf{x} + \theta_0 = \omega_0 \tau + \theta_0 \quad (34)$$

The initial phase θ_0 is arbitrarily selected and remains immeasurable in an absolute sense owing to the exceedingly high frequency ω_0 . It is noteworthy that, given the energy-frequency relation $\theta = \omega_0 \tau = \frac{E_0}{\chi} \tau$, the relativistic action assumes the following form:

$$S = mc^2 \int d\tau = E_0 \tau = \chi \theta. \quad (35)$$

The additive nature of τ , combined with the need to map back to S^1 , generates mathematical inconvenience. Hence, usage of a complex number is natural:

$$e^{i\theta} = e^{iS/\chi} \quad (36)$$

Multiplication of phases rather than addition characterizes the operation of the group $U(1)$, producing the desired additive structure and equivalence class. Incidentally, the resulting object matches the quantum phase factor, though the factor was neither postulated nor employed in the form of an ansatz—the choice arose purely from the need for a mathematical representation of an ontic phase. A soliton with a single phase frequency is a perfect clock, and accordingly the phase is interchangeable with time. The substitution proves instrumental for determining a formula for the current represented by ψ .

Two objects now characterize the soliton, both extending through all of space: $\rho(\mathbf{x}, t)$, strictly non-negative, and $\theta(\mathbf{x}, t)$. Combination of these into a single complex number respecting the domain of both proceeds naturally²⁰:

$$\psi(\mathbf{x}, t) = \sqrt{\rho(\mathbf{x}, t)} e^{i\theta(\mathbf{x}, t)} \quad (37)$$

This combination is differentiable and multiplicative, restricting ρ and θ within their respective domains and manifolds while maintaining bijectivity, **except for the case**

¹⁹A second phase variable also demands attention: the polarization between electric and magnetic components of the soliton. This is likely where the $SU(2)$ group action resides.

²⁰Other representations may be chosen and the resultant relations remain identical provided the domains of both manifolds are correctly accounted for. The mapping onto the complex plane is simply the most convenient one.

$\rho = 0$ where the phase θ is undefined. The latter property alters the outcome of any loop integral containing an undefined point or region. An additional redundancy arises from the overall phase offset, which is arbitrarily assigned a value of 0; this overall phase becomes measurable solely when it constitutes a coherent offset relative to *another* soliton's phase. Although alternative representations of ρ and θ exist, the resulting mathematics may prove inconvenient for Lagrangian formulation, particularly given that extracting the square root of the energy density yields a quantity linearly proportional to the field yet strictly positive, thereby precluding interference with the phase term. The quantity ρ is recovered by the conjugate product of ψ , and a representation of θ is obtainable by rescaling to the unit circle.

$$\rho = \psi^* \psi = |\psi|^2 \quad (38)$$

$$(\cos \theta, \sin \theta) = \frac{1}{2\sqrt{\psi^* \psi}} \left(\psi + \psi^*, \frac{1}{i}(\psi - \psi^*) \right) \quad (39)$$

Both ψ and ψ^* constitute Lorentz-invariants, representing a soliton state in ϕ . The component θ is also Lorentz-invariant, and ρ represents the stationary configuration of the soliton in the rest frame—also Lorentz-invariant. Annihilation of the soliton (with ρ vanishing everywhere) breaks the correspondence. On a historical note: the preliminary ansatz of Schrödinger, $S \approx K \ln \psi$, has been derived here by construction rather than postulation—emerging from the process of elimination and the reality of relativistic soliton motion. The question arises: what **is** ψ ? Does ψ constitute an ontic object? The answer is conditional: in the rest frame of the soliton, ψ was assembled from the field energy density of the soliton combined with the real phase by multiplication and appropriate representation. Outside the starting frame, whether ψ permits exact prediction given available physical knowledge becomes a separate question. Once ψ leaves a coherent state, ψ ceases to function ontically and becomes an *epistemic* object—physics renders ψ a coarse-grained descriptor. It will become apparent that the ontic ψ is unnecessary for physical insight. The epistemic ψ suffices for many applications, much as the ignorance of exact system states is sufficient for the theory of gases.

8.3 Factoring Out the Source Term

The object ψ represents an attempt at factoring out a portion of ϕ , generated by the potential A^μ , that possesses a nonlinearity maintaining the same shape in the rest frame, while Lorentz invariance deforms the potential with each motion. Only one field $F^{\mu\nu}$ exists, and with sufficient computational tools, perfect understanding of the complete nonlinear (and unified) field, and perfect knowledge of spatial arrangement at any instant, the analysis could end with ψ discarded and patterns extracted directly from A^μ . Such omniscience and infinite computational power remain unavailable, however, and ψ therefore persists in the role of an abstraction of matter.²¹

The quantity $\rho_q = q|\psi|^2$ represents a source term in Maxwell's equations, constituting the charge-weighted presence by definition. A current $\mathbf{J}$ readily emerges from $|\psi|$ given its construction from the stress tensor, and the continuity equation applies directly. The Hamilton-Jacobi equation in covariant form for the momentum P^μ reads:

²¹Of course, taking the nonlinear interaction involves dropping ψ and dealing with A^μ directly—in a sense, precisely what QED attempts.

$$P_\mu = \partial_\mu S = \chi \partial_\mu \theta \quad (40)$$

The velocity takes the form of the usual linear relation of momentum-velocity $P^\mu = mu^\mu$:

$$u^\mu = \frac{\chi}{m} \partial^\mu \theta \quad (41)$$

Since the current is simply the density multiplied by the velocity:

$$J^\mu = \rho_q u^\mu = q \rho u^\mu = \frac{q\chi}{m} \partial^\mu \theta \cdot \rho \equiv \frac{q\chi}{m} j^\mu \quad (42)$$

In the last part of the expression, $j^\mu \equiv \rho \cdot \partial^\mu \theta$ was defined. $\rho \cdot \partial_\mu \theta$ emerges from the object ψ by conjugation and the product rule:

$$\partial^\mu \psi = \frac{\partial_\mu \rho}{2\rho} \psi + i \partial^\mu \theta \psi \quad (43)$$

$$\partial^\mu \psi^* = \frac{\partial_\mu \rho}{2\rho} \psi^* - i \partial^\mu \theta \psi^* \quad (44)$$

$$\psi \partial^\mu \psi^* = \frac{\partial_\mu \rho}{2} - i \partial^\mu \theta \cdot \rho \quad (45)$$

$$\psi^* \partial^\mu \psi = \frac{\partial_\mu \rho}{2} + i \partial^\mu \theta \cdot \rho \quad (46)$$

Subtracting the last two terms and scaling, the current follows from (42):

$$j^\mu = \frac{i}{2} (\psi \partial^\mu \psi^* - \psi^* \partial^\mu \psi) \quad (47)$$

$$J^\mu = \frac{q\chi}{m} j^\mu = \frac{q\chi}{2m} i (\psi \partial^\mu \psi^* - \psi^* \partial^\mu \psi) \quad (48)$$

As one might expect, j^μ contains a differential of the density of charge in the rest frame as the temporal component of j^μ . Moreover, $|\psi|^2$ is the charge density and obeys continuity $\partial_\mu j^\mu = 0$ as it was assembled from the stress-energy tensor. With the tensor of the field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, Maxwell's equations become:

$$\partial_\mu F^{\mu\nu} = \mu_0 J^\nu \quad (49)$$

Done— ψ can now be thrown away... Just kidding! The preceding result is valid for a snapshot alone, or when dealing with macroscopic or high-speed free electrons. At microscopic scales, Lorentz invariance and the extended nature of the soliton come into play. Additionally, the unknown state of A^μ interacts with the soliton continuously. Mixed together, even a deterministic and real system cannot yield j^ν once time evolution begins, nor can the complete state of the system be known even at $t = 0$. The sole recourse is to deal with ψ and accept that any obtainable result merely constrains what the system can do. Localizing the charge can help determine eigenstates and their stability as the lowest-order term of a nonlinear interaction that presumably cancels out in steady state. Further discussion follows after arriving at the linearized governing equation for ψ .

A wave equation is required for further progress. Instead of guessing, what follows derives the sole possible linear form coupling correctly to Maxwell's equations.

9 The Source-Free Wave Equation

9.1 Dispersion Relation

The restorative force and Lorentz invariance have been the focus so far. It also proves useful to study the dispersion relation already discovered. Returning to figure 2, consider what type of dynamics would guide the particle. In all instances, nonlinear forces keep the particle together, even when its location is stretched out through spacetime. The soliton condition implies that it will always restore itself. Here lies the key: the soliton can be considered its own continuous medium, with a restorative force and a dispersion relation derived shortly. So long as no energy is lost from the system, it will oscillate between states or simply diffuse over space. The moment the particle radiates, the assumption fails and the system loses energy. Hence, the equation derived below handles only the steady-state of a soliton—not transitions, or in other words, not electrodynamics.

A lossless system with a restorative force and dispersion obeys a wave equation. What is the dispersion relation of the soliton? Recall the Planck energy relation from relativity²²:

$$E^2 = p^2 c^2 + m^2 c^4 \quad (50)$$

Substituting the values in equation (31), factoring out the χ , and noting that $E_0^2 = m^2 c^4 = \chi^2 \omega_0^2$ yields:

$$\omega^2 = c^2 k^2 + \omega_0^2 \quad (51)$$

The same formula follows by considering the aggregated wavenumber in the rest frame of the soliton, following a similar approach to section 7: $K_i^\mu = (\frac{\omega_0}{c}, \mathbf{0})$. Under a boost, the result becomes $K^\mu = (\frac{\omega}{c}, \mathbf{k})$. Via Lorentz invariance:

$$K_{i\mu} K_i^\mu = K_\mu K^\mu = \frac{\omega^2}{c^2} - |\mathbf{k}|^2 = \frac{\omega_0^2}{c^2} \quad (52)$$

Rearranging (52) yields the same formula as (51). It is insightful to see how this expression is approximated for non-relativistic group velocities:

$$\omega = \omega_0 \sqrt{1 + \frac{c^2 k^2}{\omega_0^2}} \approx \omega_0 + \frac{c^2 k^2}{2\omega_0} = \omega_0 + \frac{\chi}{2m} k^2 \quad (53)$$

The constant term represents the restorative force, which plays zero part in the dynamics unless energies begin approaching the total energy of the electron. Factoring the term out produces what is called the Schrödinger dispersion relation:

$$\omega = \frac{\chi}{2m} k^2 \quad (54)$$

Considering the envelope of the electron for non-relativistic speeds alone, observe the velocity of phase and calculate the connection to group velocity:

$$v_{phase} = \frac{\omega}{k} = \frac{\chi k}{2m} = \frac{p}{2m} = \frac{1}{2} v = \frac{1}{2} \frac{\partial \omega}{\partial k} = \frac{1}{2} v_{group} \quad (55)$$

²²This is readily obtained by examining the relativistic action and matching the non-relativistic relation derived in Landau and Lifshitz. The next chapter sketches this derivation.

9.2 Wave Equation from Dispersion Relation

Curiously, the non-relativistic velocity of phase is **slower** than the velocity of the group when the restorative component is removed. The result makes perfect sense. Consider a flexural membrane (1D)²³. It has the dispersion relation $\omega^2 = \frac{EI}{\rho A} k^4 = \alpha k^4$, for elastic constant E , second moment of inertia I , density of material ρ and cross-sectional area A . By applying a Fourier transform on the deflection $\eta(x, t)$, $\bar{\eta}(k, \omega)$:

$$\bar{\eta}(k, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \eta(x, t) e^{-i(kx - \omega t)} dx dt \quad (56)$$

Here k is the spatial wavenumber and ω is the angular frequency. The inverse transform is:

$$\eta(x, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{\eta}(k, \omega) e^{i(kx - \omega t)} dk d\omega \quad (57)$$

Under these transform pairs, temporal and spatial derivatives become algebraic multiplications: $\partial_t \rightarrow -i\omega$ together with $\partial_x \rightarrow ik$. Consequently, $\partial_{tt} \rightarrow -\omega^2$ and $\partial_{xxxx} \rightarrow k^4$. Nothing new here—these are basic Fourier transform pairs obtainable via integration by parts. Multiplying the dispersion relation with the transformed deflection $\bar{\eta}$:

$$\omega^2 = \alpha k^4 \Rightarrow \alpha k^4 \bar{\eta} - \omega^2 \bar{\eta} = 0 \quad (58)$$

Inverting the Fourier transform yields a wave equation from the dispersion relation:

$$\eta_{tt} + \alpha \eta_{xxxx} = 0 \quad (59)$$

The interesting thing about such an expression: flexural waves can travel at arbitrary velocity yet fail to maintain their shapes. In sharp contrast, once a nonlinear and nonlocal amplitude-dependent restorative force is introduced (for instance, boundary conditions on the ends generating an axial force), the resultant Woinowsky-Krieger PDE supports solitons:

$$\eta_{tt} + \alpha \eta_{xxxx} - \beta \left(\int_0^L (\eta_x)^2 dx \right) \eta_{xx} = 0 \quad (60)$$

The speed of these solitons is an affine function of the amplitude of the solitons, linearly related to the energy of the mode. The correspondence plays a role analogous to that of the fundamental soliton derived earlier, with amplitude and velocity analogous to energy and frequency of phase. The restoration is a consequence of high amplitude ‘deformation’ of the medium. The nonlinear ‘stiffening’ term is **nonlocal** and quadratic. The form of the term is curiously similar to the nonlocal nonlinear Schrödinger PDE used in quantum optics [39].

Taking the same route as the previous PDE and applying a Fourier transform of ψ , $\bar{\psi}$, multiplying the result by the dispersion relation that is already in Fourier space:

$$\omega \bar{\psi} = \frac{\chi}{2m} k^2 \bar{\psi} \quad (61)$$

Inverting the Fourier transform yields the source-free Schrödinger equation from the dispersion relation alone (multiply both sides by a factor χ to obtain the familiar form), and noting $k^2 \rightarrow -\nabla^2$:

²³The PDE, in the form of flexible plates, was mentioned in passing by Schrödinger in his original papers.

$$i\frac{\partial\psi}{\partial t} = -\frac{\chi}{2m}\nabla^2\psi \quad (62)$$

The result proves unsurprising—once the dispersion relation exists, the route is always available. Most pedagogical treatments typically identify the left-hand side with energy, subtract away a potential, and claim derivation of the result. Such treatments are invalid derivations, as they introduce a Sturm-Liouville-style factor in an otherwise source-free equation by arbitrarily adding a mixed potential-wavefunction term. If the pedagogues desired simplicity, they could treat the wavefunction as a medium with a position-dependent phase velocity:

$$\square_{v_p(\mathbf{x})}\psi = 0 \quad (63)$$

by noting how the equations (31) grant $v_p = E/p$, where E is the Hamiltonian $E = \frac{p^2}{2m} + V$, and by carrying out a Fourier transform (retaining the positive contributing terms alone) one can demonstrate equation (63) equals the Schrödinger equation. The reader is invited to demonstrate the equivalence for themselves. In other words, the Schrödinger equation reduces to a wave equation with the caveat negative energy solutions must be discarded to obtain the eigenvalues and eigenstates.

These “derivations,” fail to constitute a legitimate derivation. The next section arrives at the same result via a constructive treatment adhering to greater rigor, legitimately built from foundational axioms.

Towards that aim, the restriction on non-relativistic speeds must be removed. Repeating the derivation of (62) emerges, with the relativistic dispersion relation (51), and multiplying both sides by a Fourier transformed wavefunction $\bar{\psi}$:

$$\omega^2\bar{\psi} = c^2k^2\bar{\psi} + \omega_0^2\bar{\psi} \quad (64)$$

Remembering the previously mentioned Fourier relations:

$$-\frac{\partial^2\psi}{\partial t^2} = -c^2\frac{\partial^2\psi}{\partial x^2} + \omega_0^2\psi \quad (65)$$

Rearranging and substituting the energy-frequency relation $\omega_0 = \frac{E_0}{\chi} = \frac{mc^2}{\chi}$:

$$\frac{\partial^2\psi}{\partial t^2} - c^2\frac{\partial^2\psi}{\partial x^2} + \left(\frac{mc^2}{\chi}\right)^2\psi = 0 \quad (66)$$

Dividing by c^2 produces the form of d'Alembert:

$$\boxed{\square\psi + \frac{m^2c^2}{\chi^2}\psi = \left(\partial_\mu\partial^\mu + \frac{m^2c^2}{\chi^2}\right)\psi = 0} \quad (67)$$

The derivation yields the source-free Klein–Gordon equation²⁴. The derivation above needs zero postulates and zero operator formalism, just the dispersion relation and a Fourier transformation. Most derivations introduce operators and postulate the energy-frequency Planck relation and the de Broglie relation. Such steps are absent here. The PDE represents the restorative nonlinear electromagnetic force of the soliton configuration

²⁴Technically, calling the equation the Fock equation after the Soviet physicist Vladimir Fock who derived the equation before the two after him remains appropriate, although often in science things remain named after the latest person to ‘discover’ them rather than the first one.

tied to energy circulation. The equation comprises Lorentz-invariant forms only, which should not be too surprising since the Planck energy-momentum relation is the only possible Lorentz-invariant relationship. The next chapter confirms the uniqueness of such derivation by considering all possible local terms in the equivalent Lagrangian.

The PDE can only express the dynamics of a lone soliton in an empty universe exclusively. Considered in isolation, this result holds little interest. Yet it *is* the source-free Klein–Gordon equation. The next step is embedding the soliton in the physical universe. Achieving that involves a return to Maxwell’s equations, properly coupling ψ to A^μ and through a maximal approach.

10 Coupling Electromagnetism to the Wavefunction

Now the point has been reached where everyone had given up or already smuggled in the usual postulates. Or reached for operators and smuggled in postulates without realizing the smuggling. The present work avoids these traps. Up until now the math has been kept relatively light, but the next section assumes some knowledge of symplectic geometry. *Mechanics* of Landau and Lifshitz together with *Calculus of Variations* of Gelfand and Fomin offer solid grounding. What follows succeeds in embedding the soliton together with the outer electromagnetic field utilizing physics and mathematics developed in the 1800s.

10.1 Formulations of Lagrangians

Extracting dynamics directly from continuum conditions while maintaining Lorentz invariance is possible but extremely delicate. Dealing with energies or the total action of the system is far more practical. The subsection provides a self-contained explanation of Lagrangian mechanics; readers familiar with the topic may skip to the next subsection. In the mid-1700s, Pierre Louis Maupertuis [40] brilliantly realized the laws of Newton could be expressed as a problem of minimisation. He defined an action as the ‘effort’ required by systems to create motion, the product of mass, velocity and distance. This possesses the unit of time multiplied by energy.

Euler, Lagrange, Hamilton and Jacobi later refined this idea to very general systems, but all these methods share a matching underlying mechanism: The target system will always take the course that finds stationary action²⁵. Once accumulation of action is defined (through a Lagrangian), the dynamics follow directly.

The approach proceeds as follows, defining an action:

$$S = \int_0^T L dt = \int_0^T (K - V) dt \quad (68)$$

Where K denotes kinetic energy and V potential energy within the system. By finding a stationary path on S in the configuration space of the system described by L , a PDE may be found. Applying the Euler-Lagrange relation (derived from the calculus of variations) for every generalized coordinate (q_i) and the derivative of the coordinate ($\dot{q}_i$) yields:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) = \frac{\partial L}{\partial q_i} \quad (69)$$

Lorentz invariance is a constraint on which terms can appear in the Lagrangian—all terms must be invariant or physical dynamics would depend on choice of inertial frame. For instance, how was equation (35) obtained? One way: adopt the route of Planck, calculate the momentum and construct the action directly (momentum integrated over the spacetime interval). Another route: construct the action through process of elimination, illustrated in Landau and Lifshitz [41]. The second route will prove fruitful.

Consider a soliton in the world of the soliton, which could be localized at position $x(\tau)$ where τ is the proper time of the soliton. It is characterized by an aggregated position in configuration space, its mass, charge and rest frequency. Since all of these are constants, they are lumped together as a factor α awaiting determination. The action in the rest

²⁵This variational problem admits framing as a minimization or maximization depending on sign choice, alternatively a saddle point for higher order Lagrangians.

frame of the soliton must match that measured by an observer moving at speed v . The most general expression of the action S must be:

$$S = -\alpha \int_a^b ds = -\alpha c \int_a^b d\tau = \int_a^b L dt \quad (70)$$

The negative sign is chosen because the spacetime interval is maximized when a particle is stationary and finding a minimized stationary solution is conventional, though the sign would drop out at the end as a stationary solution anyway. The mathematical identity whereby the spacetime interval in the rest frame of the particle and inertial frame is simply the proper time scaled by speed of light c , was also used. How are t and τ related? By the γ boost:

$$dt = \gamma d\tau \quad (71)$$

In sharp contrast to the phase frequency of the soliton which speeds up upon boosting, the ‘proper time’ of the particle in the kinematic formulation of relativity *slows down*.

The Lagrangian can be pulled out as the integrand in the configuration space where the soliton is moving:

$$L = -\alpha c \frac{1}{\gamma} = -\alpha c \sqrt{1 - \frac{v^2}{c^2}} \quad (72)$$

Then, taking the non-relativistic limit to find α , with $v \ll c$. Since the Lagrangian is potential-free, it shall contain a velocity-dependent term representing the kinetic energy up to a constant:

$$L = T \approx -\alpha c \left(1 - \frac{1}{2} \frac{v^2}{c^2} \right) = -\alpha c + \frac{\alpha v^2}{2c} \quad (73)$$

The first term is constant and thus cannot contribute to the dynamics. The second term yields Newtonian kinetic energy, $\frac{1}{2}mv^2$, hence $\alpha = mc$ and the Lagrangian becomes:

$$L = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} \quad (74)$$

Now, a Legendre transformation is applied to extract a Hamiltonian and the canonical momentum. In turn, this yields the energy-momentum Planck relation previously used (equation 50). That formulation is an exercise for the keen reader. The key point: demonstrating a Lagrangian admits construction by considering all permissible terms, provided real physical restrictions are applied, yields the physics of a system.

In a sense, the Lagrangian is being constructed in the negative space, through admission of *any* possible term. The coefficients are then determined through experimental measurements, in the case examined, non-relativistic behavior. The key insight enables deriving the sourced Klein–Gordon equation from scratch in the section to come.

10.2 Bilinear Lagrangian of EM and Soliton Fields

The next task of the derivation is coupling the field generated by A^μ , $F^{\mu\nu}$, to the complex potential ψ . The most important conceptualization is reiterated: ψ is in fact a part of a unified electromagnetic field containing $F^{\mu\nu}$. It is conceptually partitioned in the following manner:

$$\boxed{\Phi^{\mu\nu} = F^{\mu\nu} + \Sigma_i S_e[\psi_i]} \quad (75)$$

In this conceptual equation, $S_e[\psi_i]$ materializes the actual electric and magnetic fields of the electron-type soliton indexed by i in the unified field Φ ²⁶. The resulting nonlinear electromagnetic field generates a restorative force maintaining the integrity of the solitons represented by ψ_i . In the absence of any solitons, Φ and F assume the same value, yet their dynamics differ entirely: Φ contains an entirely unknown Lorentz-invariant field interaction. In a sense, all available knowledge of the electron has been encoded in ψ , and only a coarse-grained view of its interaction together with the remainder of the field Φ remains accessible. Since all solitons (and all types of solitons) can be summed in this manner, it should occasion no surprise that matter has been observed to appear, disappear, and exhibit entanglement across great distances. Nor should it prove surprising that the same Lorentz invariance appearing to hold for light also holds for ‘matter’: the sole existent entity is the universal field Φ .

This picture is merely illustrative, but useful. The dynamics behind Φ are unknown and possibly highly nonlinear, but linearization within each epistemic field representation is possible while maintaining an isolated **bilinear** interaction between $F^{\mu\nu}$ and a soliton $S_e[\phi]$. These would then interact in the following abstract manner, with a dynamic operator L over each of the fields and additive and coupling operators:

$$L[\Phi] \approx L_f[F^{\mu\nu}] \oplus L_e[S_e[\psi]] \oplus L_i[F^{\mu\nu} \otimes S_e[\psi]] \oplus (\text{higher orders}) \quad (76)$$

The L operation functions analogously to Fourier transformation: high-frequency (soliton) terms separate from low-frequency matter-free field terms, and nonlinear products are reduced to single bilinear coupling terms. To re-emphasize, given Φ and its Lagrangian, this separation would become unnecessary. Nonlinearity in higher-order field interactions generates infinite entanglements between the field and each soliton within the field. Such entanglement is intrinsic, not bolted on later through tensor products over Hilbert spaces. Linearity holds in both domains: wavefunctions superpose, Maxwell fields add—yet analyzing domain-crossing physics remains impossible without the higher-order unified field. Bilinear maps between spaces X and Y are linear in each argument separately:

$$B(a_1x_1 + a_2x_2, y) = a_1B(x_1, y) + a_2B(x_2, y), \quad (77)$$

$$B(x, a_1y_1 + a_2y_2) = a_1B(x, y_1) + a_2B(x, y_2) \quad (78)$$

Now how can any of the terms in (76) possibly be known, given that:

- The electromagnetic structure of the soliton is unknown—only that the soliton possesses a phase frequency, possibly a polarization representing spin, a certain mass, a Coulomb field, etc.
- The nonlinear interaction or ‘operators’ are unknown.
- Whether the linearized solution constitutes a good description of reality is unknown.

²⁶High-field electron-electron interactions have been neglected; these may elastically deform solitons as they pass through one another. The equation is purely illustrative. Moreover, solitons and topologies beyond electrons surely exist, so this does not constitute a truly unified field—though it suffices for present purposes.

Here, the ontic picture solves the problem. Nothing is postulated; only what is known empirically and what has been derived so far is employed. To lay the groundwork, consider the Lagrangian already established before answering these questions within the coming subsection.

What is *known*: In the operation L_f , matter has been factored out of the field, setting $j^\mu = 0$. An effective field emerges, though without matter. Similarly, in the operation L_e , no potential field exists—just an electron with the dispersion relation derived earlier. The evolution of both parts is known exactly: Maxwell’s equations and the source-free Klein–Gordon equation derived earlier. Yet L_e fails to completely describe the soliton, as the soliton will experience a nonlinear and likely nonlocal restorative force characterized by the higher orders of $L[\Phi]$.

Each of these can be expressed in terms of a Lagrangian density $\mathcal{L}$. For Maxwell’s equations, it is well known that the sourced Lagrangian is a function of the field, the current density and the vector potential:

$$\mathcal{L}_{f,\text{sourced}} = -\frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} - A_\mu J^\mu \quad (79)$$

The A_μ factor renders the expression a coupling term, ensuring J^μ survives variation, but the potential itself is epistemic, not ontic. How might the Lagrangian be expressed in terms of ontics alone? No local Lagrangian suffices—none in which energy depends only on the canonical coordinate value being integrated over. This detour demands moving up to the action itself and integrating over all currents. A non-causal kernel²⁷ yields what may be the most beautiful and symmetrical expression in all of physics:

$$S_f = -\frac{\mu_0}{8\pi} \int \int d^4x J_\mu(x) \delta((x - x')^2) J^\mu(x') d^4x' \quad (80)$$

The 8-dimensional integration expresses the full connection between all current densities through all of space and time, in all directions, past and future. The integral yields a charge-only ontology but possesses an equivalent field formulation. To restore the fields, the Lorenz gauge (introduced shortly) is employed to express J in terms of A^μ and to minimize the action, whereupon Maxwell’s equations and Lorentz invariance emerge. This constitutes a continuous auto-bilinear process—in a sense, the maximal one achievable across the entirety of spacetime. For instance, if spacetime is discretized (or ‘quantized’ in QED parlance) and transformed into a finite block with indices i, j , and the Green function $-\frac{\mu_0}{8\pi}\delta((x - x')^2)$ is reconstructed with a large, fixed matrix K_{ij} , the integral becomes a bilinear matrix operation:

$$S_f = \sum_{i,j} J_\mu(x_i) K_{ij} J^\mu(x_j) \quad (81)$$

The Schrödinger equation can readily be reached from (80), but doing so would introduce freedom of choice, which is antagonistic to a postulation free methodology. What is worse is that the action expression does not support free waves without an additional boundary condition, so all waves are charge-to-charge mediated. Therefore, retaining A^μ is necessary henceforth.

Factoring out the charge-potential coupling term $j^\mu A_\mu$ to derive the free Maxwell Lagrangian:

²⁷The full-field Wheeler–Feynman action.

$$\mathcal{L}_f = -\frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} \quad (82)$$

Note: while the expression is free of the direct value of A^μ in this form, $F^{\mu\nu}$ is built from the curl of the very same potential:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (83)$$

Likewise, the coupling term in Maxwell's sourced Lagrangian density depends directly on A^μ (a generating potential). Just as an integral is not uniquely defined by its derivative, the physical field is not uniquely identified by its generating potential A^μ —gauge freedom exists. Any function $\partial^\mu \lambda(x^\mu)$ may augment A^μ , and the Lagrangian will still produce the exact same dynamics provided current is conserved ($\partial_\mu j^\mu = 0$). A solution to this Lagrangian that still contains gauge freedoms is:

$$\square A^\mu - \partial^\nu (\partial_\nu A^\mu) = 0 \quad (84)$$

A natural choice of gauge is implied: $\partial_\mu A^\mu = 0$, the Lorenz gauge²⁸, which is Lorentz-invariant. Should that gauge be chosen, the expression for A^μ reduces to the wave equation:

$$\square A^\mu = 0 \quad (85)$$

Since these are the same wave equations from which Lorentz transformations were extracted as a symmetry group, the equations share that symmetry group. Lorenz chose this gauge because it decoupled the electrostatic potential and magnetic vector potential in the wave equation. This choice will illuminate a result to come; the reader is advised to bear it in mind. The source-free Klein–Gordon equation has a form very similar to the wave equation for the potential field but exhibits a potential term (mass):

$$\square \psi + \frac{m^2 c^2}{\hbar^2} \psi = \partial_\mu \partial^\mu \psi + \frac{m^2 c^2}{\hbar^2} \psi = 0 \quad (86)$$

How does one express this as a Lagrangian? Consider the Euler-Lagrange equation in reverse: what Lagrangian would produce such dynamics? Following §32 of Landau & Lifshitz [41], the Lagrange density $\mathcal{L}(\psi, \psi^*, \partial_\mu \psi, \partial_\mu \psi^*)$ generates a generalised form of the Euler–Lagrange equation:

$$\frac{\partial}{\partial x^\mu} \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi)} \right) - \frac{\partial \mathcal{L}}{\partial \psi} = 0 \quad (87)$$

with an identical result for ψ^* . The complete Lagrangian can have a strictly constant multiplicative complex phase (which would factor out), but it cannot have a differing complex phase between the terms or the variational principle employed to derive the preceding result no longer works. To simplify things, without loss of generality, require a real expression for the Lagrangian.

The covariant form of the Klein–Gordon equation makes it easy to see what the Lagrangian should be, but pay attention to the conjugation:

²⁸Not a typo: Lorenz (Danish) is not the same person as Lorentz (Dutch); he lived in an earlier era. Quite a coincidence — many physicists get that wrong, even Landau and Lifshitz. See equation 46.10 in the fourth revised English edition of the classical theory of fields book.

$$\mathcal{L}_e = \partial^\mu \psi^* \partial_\mu \psi - \frac{m^2 c^2}{\chi^2} \psi^* \psi = \frac{1}{c^2} |\partial_t \psi|^2 - |\nabla \psi|^2 - \frac{m^2 c^2}{\chi^2} |\psi|^2 \quad (88)$$

The conjugation guarantees that the Lagrangian is real. Additionally, the conjugation generates two identical results for ψ and its conjugate, which is consistent. Note that the Euler–Lagrange equation for ψ produces a conjugate result for ψ^* , and the counterpart equation for ψ^* produces a result for ψ . For instance:

$$\frac{\partial \mathcal{L}}{\partial \psi^*} = -\frac{m^2 c^2}{\chi^2} \psi, \quad \frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi^*)} = \partial^\mu \psi \quad (89)$$

Though ψ and ψ^* are independent variables in the Lagrangian (because the variables are derived from two independent variables ρ and θ), they represent one whole particle. For that to make sense, the equations of motion (hereafter EOM) must match upon conjugation; hence, any terms added must be Hermitian²⁹. What does that restriction entail? Is the source-free Klein–Gordon equation derived here indeed unique for the soliton ontologized? Perhaps Lorentz invariance and the fundamental soliton still permit a higher degree of measurable dynamics without requiring nonlinear mixing. Consider all linearly independent Lorentz-invariant Hermitian terms that might be added while keeping the Lagrangian real:

- α : A constant value only shifts the Lagrangian and does not change the variational solution of the action— $\alpha = 0$ may be selected without loss of generality, still yielding the same EOM.
- $\beta \sqrt{\rho} \cos(\theta) = \frac{\beta}{2}(\psi + \psi^*)$: This term leaves behind a constant factor in the variational problem, yielding a Klein–Gordon equation with a constant source term. The result renders localization of the soliton impossible, since ρ maintains a minimal density everywhere and does not tend to zero as the radius approaches infinity. The term therefore contradicts the physical situation and must be discarded: $\beta = 0$. The same reasoning applies to the $\tilde{\beta} \sin(\theta)$ term, so duplication is avoided for brevity—should the $\cos(n\theta)$ case be eliminated, the $\sin(n\theta)$ cases follow likewise.
- $\frac{\epsilon}{2}(\psi^2 + (\psi^*)^2)$: This case proves interesting. The term couples the equations of motion for ψ and ψ^* , yielding what appear to be two particles with differing effective masses due to a conjugate dependency in the lower order. Fourier transforming both resulting EOMs produces two frequencies governed by two dispersion relations. The coupling could additionally serve as a parameter indicating soliton decoherence and disintegration—implying instability—yet such behavior is not observed for electrons. In summary, this addition breaks the single energy-phase frequency formulation established earlier together with the Planck energy-momentum relation determined from Lorentz invariance. Hence, $\epsilon = 0$.
- $\frac{\eta}{2}(\partial_\mu \psi \partial^\mu \psi + \partial_\mu \psi^* \partial^\mu \psi^*)$: The same issue arises as in the previous case: two effective dispersion relations emerge, and the term thus no longer represents one whole particle.
- $\lambda \psi^* \psi (\psi + \psi^*)$ and other higher-order terms: These produce a nonlinear term $|\psi|^2$ in the wavefunction, associated with higher-energy (or possibly higher-frequency)

²⁹For readers unfamiliar with the term: this condition requires $a^* = a$.

interactions of the unified field Φ . The aim here is to determine the linear EOM for low-energy field–single-soliton interaction (well below the Schwinger limit), so these and higher-order terms are excluded as belonging to higher-order nonlinear interactions. The goal of finding a **bilinear** interaction precludes leaving nonlinear terms in any of the coupled systems. This constitutes a limitation: two interesting cases exist where this assertion may fail. The first arises when a soliton dynamically transitions between frequencies (acceleration or ‘absorption and emission’), which exhibits strong temporal dependency. The second occurs when an electron passes through an atomic nucleus and momentarily experiences immense fields. Nevertheless, these effects remain higher-order interactions, and the present goal is to determine the unique lowest-order consistent EOM as a preliminary step. Extension to higher-dimensional terms would demand systematic order-by-order analysis incorporating other solitons and soliton types. The present treatment restricts itself to a lone soliton and a low-energy field. Hence, $\lambda = 0$.

- As usual with Lagrangian derivations, higher derivatives of ψ and ψ^* are avoided due to the Ostrogradsky instability, which would render them nonphysical with ghost canonical variables unless additional structure is introduced at higher levels. Inclusion of such terms could also be ruled out experimentally, shortening the search space accordingly. This restriction should not preclude higher-order theories, and solitons may **need** to violate this rule, as seen in KdV and the flexural beam equation. Yet the dimensional freedom to achieve this here is simply absent. A recent paper on this subject [42] shows promising progress in the theoretical treatment of these terms for potential future extensions.
- Finally, according to the best available experiments, the medium of propagation is isotropic and homogeneous in the absence of matter, so no x^μ terms appear anywhere within the Lagrangian.

For completeness, the question arises: what about nonlinear functions of ψ and ψ^* ? Such functions could be Taylor-series expanded and their terms distributed into lower orders. What about *nonlocal* integrals involving the coordinates in the Lagrangian across all of spacetime, such as bilocal kernels? That approach meets its limit here. Such terms constitute another class of functional operator, one yielding an infinite search space regardless of the target Lagrangian’s order. A crucial point: as will eventually become clear, the true ontic Lagrangian must be nonlocal. The goal here is far humbler than solving that cosmic mystery — simply obtaining the Schrödinger equation without postulates, learning what emerges along the way.

Returning to the previous point, how is ‘low order’ determined? By borrowing an idea from dimensional analysis. The entire Lagrangian carries units of energy, even if $\mathcal{L}$ carries units of energy density. When uncoupled, an arbitrary scaling factor may be applied. Upon coupling, however, ψ acquires a unit of the square root of charge density, which complicates classification; the value 1 is taken for consistency. The soliton relations ($E = \chi\omega$, $p = \chi k$, hence $E/p = \omega/k$) place inverse length and inverse time on equal footing with energy when the action is factored out into the Lagrangian. These relations indicate that every derivative gains an order (as derivatives carry an inverse of these units in the denominator of the defining limit). Similarly, A^μ is the vector potential and therefore has units of energy once coupled with charge. By this method of dimensional analysis, the following order emerges for every term:

$$[A^\mu] : 1, \quad [\psi] : 1, \quad [\partial_\mu] : 1, \quad [F = \partial_\mu A^\nu - \partial_\nu A^\mu] : 2 \quad (90)$$

The Maxwell Lagrangian has a dimensional order of 4 according to this scheme. That order defines low-energy interaction and serves as the cutoff chosen for the interactive terms derived next. Is this a unique choice? No upper limit exists, but if ‘low order’ is defined as ‘same order as Maxwell’s equations,’ then the procedure can stop at exactly 4, leaving experimental consistency to be verified. This does not necessarily imply that everything excluded is nonlinear: some of the nonlinear terms already eliminated have order less than 4. Yet the cutoff halts the combinatorial explosion that would otherwise overwhelm the search and analysis. As the dimension of charge has no concrete definition, the approach remains mainly heuristic. What must be demonstrated is that no further terms exist after finding a collection that satisfies all constraints on the system without over-constraining it to the point where only trivial solutions remain possible. With this established:

The source-free Klein–Gordon equation is the unique linear, local, stable, low-energy, source-free equation that obeys the soliton dispersion relations $E = \chi\omega$, $p = \chi k$ and $E^2 = p^2 c^2 + m^2 c^4$ when the field vanishes everywhere.

A major result follows³⁰, though unsurprising given the construction. Adding any terms would, intuitively, violate the relation derived from Lorentz invariance alone. Here, however, reliance falls not on intuition but on absolute mathematical certainty. This proves the Fourier technique was legitimate. Two unique Lagrangian densities now exist for the Maxwell field equation and the source-free Klein–Gordon equation derived earlier, but these are not especially interesting: they can only represent field dynamics without a soliton, while the soliton wave equation cannot include a field. Regardless, something useful emerges. If both Lagrangians are added together and scaled by factors κ_1 and κ_2 with appropriate units:

$$\mathcal{L}' = \kappa_1 \mathcal{L}_f + \kappa_2 \mathcal{L}_e \quad (91)$$

minimizing $\mathcal{L}'(A^\mu, \partial_\mu A_\nu, \psi, \psi^*, \partial_\mu \psi, \partial_\mu \psi^*)$ with respect to each configuration variable³¹ yields two completely decoupled systems behaving exactly as before: Maxwell’s source-free equation and the source-free Klein–Gordon equation. Essentially, an EOM for the combined terms $L_f[F^{\mu\nu}] \oplus L_e[S_e[\psi]]$ of the dynamic unified field equation (76) has been obtained. Simply accounting for the energies in the Lagrangian sufficed to achieve that humble aim. That leaves the final stretch: finding how to combine these two Lagrangians to also obtain a low-order approximation of $L_i[F^{\mu\nu} \otimes S_e[\psi]]$.

10.3 Maximal Soliton-Electromagnetic Field Coupling

The operator L_i is derived through the same brute-force method[43] employed for the proof of low-energy nonlinear uniqueness.³² All mixed terms of A^μ , ψ , and ψ^* are sought, with the idea that mixing (coupling) arises from a higher-order (complete) unified field theory where the abstraction of separating the soliton from the field becomes unnecessary.

³⁰Further effort could strengthen the statement, but this suffices for the sections to come.

³¹A reminder that A_ν denotes a vector packing 4 variables and $\partial_\mu A_\nu$ denotes a tensor packing 16 variables.

³²Credit is due to Julian Schwinger [43] for the original idea behind the phenomenological Lagrangian method independently formulated here. A substantially different approach is taken, however.

As before, the search is restricted to terms up to dimension 4. Each combination is examined consecutively, drawing physical or phenomenological conclusions on whether it should be included. The collection of mixed Lorentz-invariant monomials that can participate in the Lagrangian is now enumerated. Since both A^μ and ψ must appear, and A is Lorentz covariant, inclusion can occur only if (a) A matches its upper index with another A bearing a lower index, (b) ψ is differentiated (gradient) and matches that index, or (c) A is differentiated (divergence) and matches itself, then combined with ψ . The minimal dimension is thus 3.

10.3.1 Monomials of Dimension 3

Only 6 possible monomials exist with dimension 3, and 10 possible monomials with dimension 4. Examining the dimension-3 monomials reveals what can be learned from them:

$$\{A_\mu A^\mu \psi, \quad A_\mu A^\mu \psi^*, \quad A^\mu \partial_\mu \psi, \quad A^\mu \partial_\mu \psi^*, \quad \partial_\mu A^\mu \psi, \quad \partial_\mu A^\mu \psi^*\}$$

None of these can be used alone; they must be combined to create real values for the Lagrangian.

Consider the terms $\frac{\alpha}{2} (A_\mu A^\mu (\psi + \psi^*)) = A^2 \rho \cos(\theta)$ and $\frac{i\alpha}{2} (A_\mu A^\mu (\psi - \psi^*)) = A^2 \rho \sin(\theta)$ —these create the same inhomogeneous conditions as before, though a gauge choice on A can remove them (keeping in mind the only ontic object is F). A brief discussion of the latter follows. Is dependence on a gauge acceptable? One may cite the Aharonov–Bohm effect[44] as a reason to depend on A , but that interpretation is mistaken—either the assumption is made that an arbitrary choice *physicists*³³ make about the redundant term of A can affect a measurement, or ontological nonlocality is embraced [45]. Nonlocality has not only been embraced here but has been taken to its logical conclusion with the world-tube formulation. Epistemic A gauge invariance can therefore be enforced to recover ontic physics, yielding a useful theory. A gauge choice should never change the motion of a particle.

Therefore, the class of all transformations $A \rightarrow A + \partial_u \lambda$ must produce the same physics, though it may produce varying coordinates depending on the Lagrangian. If that seems confusing, recall what A originally represents: a means of making Maxwell’s equations easier to solve. A magnetic vector potential $\mathbf{A}$ may be chosen to match a magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$. A gradient can operate on any scalar function and then augment $\mathbf{A}$ without modifying the magnetic field. The situation resembles differentiating a constant value and a function—the mapping is many-to-one, and the constant value can never affect the answer. That is not necessarily a disadvantage; it is what inspired Lorenz to transform the ugly coupled potential wave equation (84) into (85). Gauge freedom is useful to physicists, but it can also obstruct the creation of a nonlocal theory. Today, physicists have taken gauge freedom to an extreme and defined epistemology through gauges, symmetries, groups, and sometimes even ‘broken symmetries’ as the fundamental mediators of forces. That approach will not be taken in this derivation.

³³If the undertone is not apparent, the argument is spelled out: a *reductio ad absurdum*. The Standard Model and QED encode global connectivity information using gauge equivalence classes of connections, but these equivalence classes are elements of the formalism, not ontological objects. Gauge theories of this kind insist on locality for point-like particles but nonlocality for gauges—which is, to put it politely, jarring.

Instead, one gauge freedom left behind in the definition of the soliton representation is revisited: the θ_0 gauge (equation 34). The physics of the coupled equations being derived should never depend on the choice of θ_0 . That is not a benign symmetry, since the soliton behaves like a perfect clock in its rest frame, and the phase represents time—meaning this symmetry is equivalent to time-translation symmetry, i.e., energy conservation.³⁴ This creates tension with A gauge freedom, resolved below without resorting to unnatural postulates. This case is no different from the Lorenz gauge but in reverse— θ was devised just like A with a redundant identifier that only makes sense in multiple-soliton scenarios, where only the difference between coherent phases is physical. The gauge level itself should never participate in the physics. Symmetries should become apparent from the ontology and its necessary modeling; ontology should never emerge from symmetries except to check the consistency of a model.

Now, any combination of monomials sensitive to this overall phase shift cannot participate in the Lagrangian—that is, any non-derivative function of θ falls out. This proves that all six dimension-3 monomials are eliminated together. This analysis could have been performed for the uniqueness proof, but the value of that discussion lies in its independence from any such enforcement. Many nuisances have now been decoupled, but something has also been somewhat lost in the process: this Lagrangian is not sensitive to the very high-frequency phase, and thus not to the nonlinear physics contained within. These terms would not have helped anyway, as they create an inconsistency involving distinctions between starting phases. For that reason, at least thus far, the theory remains unique up to this gauge freedom.

With every dimension-3 monomial eliminated, a harder problem arises: the dimension-4 monomials.

10.3.2 Monomials of Dimension 4

All 10 mixed monomials of dimension 4 are now constructed, and those with θ dependency are eliminated—namely, anything with a ψ unmatched by a ψ^* .

$$\left\{ \begin{array}{l} A_\mu A^\mu \psi^2, A_\mu A^\mu (\psi^*)^2, A_\mu A^\mu \psi \psi^*, (\partial_\mu A^\mu) \psi^2, (\partial_\mu A^\mu) (\psi^*)^2, \\ (\partial_\mu A^\mu) \psi \psi^*, A^\mu \psi \partial_\mu \psi, A^\mu \psi \partial_\mu \psi^*, A^\mu \psi^* \partial_\mu \psi, A^\mu \psi^* \partial_\mu \psi^* \end{array} \right\}$$

Of these, only the following terms adhere to the θ gauge invariance requirement, or can be combined to become insensitive to θ :

$$\{ A_\mu A^\mu \psi \psi^*, (\partial_\mu A^\mu) \psi \psi^*, A^\mu \psi \partial_\mu \psi^*, A^\mu \psi^* \partial_\mu \psi \}$$

Here are the combinations that yield real values insensitive to θ , the last of which should be familiar:

$$\{ A_\mu A^\mu \psi \psi^*, (\partial_\mu A^\mu) \psi \psi^*, A^\mu (\psi^* \partial_\mu \psi + \psi \partial_\mu \psi^*), i A^\mu (\psi^* \partial_\mu \psi - \psi \partial_\mu \psi^*) \}$$

Now observe that:

$$A^\mu (\psi^* \partial_\mu \psi + \psi \partial_\mu \psi^*) = A^\mu \partial_\mu (\psi^* \psi) \quad (92)$$

and:

$$A^\mu \partial_\mu (\psi^* \psi) + (\partial_\mu A^\mu) \psi \psi^* = \partial_\mu (A^\mu \psi \psi^*) \quad (93)$$

³⁴QFT circles call this “global U(1) symmetry,” but here it is imposed by the construction itself, not by choice or decree.

The third remaining monomial is rewritten as:

$$A^\mu(\psi^* \partial_\mu \psi + \psi \partial_\mu \psi^*) = \partial_\mu(A^\mu \psi \psi^*) - (\partial_\mu A^\mu) \psi \psi^* \quad (94)$$

Then, observe that the soliton's wavefunction ψ is normalized, implying it will generally tend to zero as infinity is approached. The action integral (68) must integrate across all 4 dimensions. Since the $\partial_\mu(A^\mu \psi \psi^*)$ term is a divergence, applying Stokes' theorem yields a surface integral with $A^\mu \psi \psi^*$ as the integrand, contributing zero to the action. A full and formal mathematical treatment could be undertaken, but simply observing that ψ must be smooth, differentiable, and localized due to the soliton's restorative force suffices—any argument against rapid convergence to zero is not physical but purely formal. The third term is therefore not independent of the second and can be dropped³⁵.

Now just three candidates remain.

$$\{ A_\mu A^\mu \psi^* \psi, (\partial_\mu A^\mu) \psi^* \psi, i A^\mu (\psi^* \partial_\mu \psi - \psi \partial_\mu \psi^*) \}$$

The third candidate is none other than a scaled current j_μ from (48) multiplied by the field A^μ —quite fortunate, as this is the old term in the sourced Maxwell equation. That will help define a scale making the soliton charge correspond to macroscopic electrodynamics later. Two *additional* terms exist, and the consistency of their presence must be determined.

When this Lagrangian derivation began, the coarse density ρ was removed from Maxwell's equations, and with it the current $\mathbf{J}$; these were then represented by ψ —the underlying soliton with its dispersion relation defined by the energy-momentum-frequency relations derived from Lorentz invariance. Restoration now proceeds by solving the A^μ -dependent variational problem to derive Maxwell's EOM (the Klein–Gordon equation will be the other, coupled system). The Lagrangian forms narrowed down are reiterated:

$$\mathcal{L}_f = -\frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} \quad (95)$$

$$\mathcal{L}_e = \partial^\mu \psi^* \partial_\mu \psi - \frac{m^2 c^2}{\chi^2} \psi^* \psi \quad (96)$$

$$\mathcal{L}_{\text{int}} = g_1 \frac{i}{2} A^\mu (\psi^* \partial_\mu \psi - \psi \partial_\mu \psi^*) + g_2 A_\mu A^\mu \psi^* \psi + g_3 (\partial_\mu A^\mu) \psi^* \psi \quad (97)$$

where conveniently scaled parameters have been selected without loss of generality, keeping a term similar to j^μ from (47). The term $\mathcal{L}_f$ is correctly energy-scaled, but the new terms are not, so they are scaled by an overall coefficient κ and both systems added together:

$$\mathcal{L} = \mathcal{L}_f + \kappa (\mathcal{L}_e + \mathcal{L}_{\text{int}}) \quad (98)$$

To simplify matters, $\mathcal{L}_f$ is harmonized into expressions of the vector potential A^μ , recalling that $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. After some algebraic manipulation and extensive calculus, the resultant field Lagrangian is obtained:

$$\mathcal{L}_f = -\frac{1}{2\mu_0} \left[(\partial_\mu A_\nu)(\partial^\mu A^\nu) - (\partial_\mu A^\mu)(\partial_\nu A^\nu) \right] - \frac{1}{2\mu_0} \left[\partial^\nu (A_\nu \partial_\mu A^\mu) - \partial_\mu (A_\nu \partial^\nu A^\mu) \right] \quad (99)$$

³⁵Or vice versa. Whichever is chosen, the pair will contribute the same value when the Euler–Lagrange equation is used.

Can the boundary condition on the last term be used to remove it from the Lagrangian? Not quite—at least not without assuming something about the variation problem. The soliton was localized, so proving that the boundary values converge to zero is trivial with a bit of formalism. To do the same for $F_{\mu\nu}$ is to assert a fall-off of the field at infinity within the universe. If a finite extent of the universe is accepted, that is fine, but that constitutes an additional postulate. Luckily, such a strong constraint is entirely unnecessary. As the formulation was local from the start, variation can occur within some compact support (say, the boundary of a lab), with the *variation* (rather than the field) being set to zero at the boundary, whereby the term is killed. The approach resembles holding both ends of a string down and then analyzing it so that what happens beyond those ends can be ignored—another artifact of formulating a local Lagrangian. Upon imposition of this assumption, the Maxwell equation contributing term is recovered³⁶, written in terms of A^μ : $\partial_\mu F^{\mu\nu} = \partial_\mu \partial^\mu A^\nu - \partial^\nu (\partial_\mu A^\mu)$.

The terms inside $\mathcal{L}_{\text{int}}$ that contribute to Maxwell's EOM are all the A^μ -related terms. The g_1 term in particular is a simple derivative of A^μ , contributing $g_1 j^\mu$. That actually equals the current from (42), but scaled. Two other terms that drop out of the Lagrangian are $2\kappa g_2 A^\mu \psi^* \psi$ and $\kappa g_3 \partial_\mu (\psi^* \psi)$. Putting them all together, the new Maxwell equation reads:

$$\partial_\mu F^{\mu\nu} = g_1 j^\nu + 2g_2 A^\nu \psi^* \psi + g_3 \partial_\nu (\psi^* \psi) = g_1 j^\nu + 2g_2 A^\nu \rho + g_3 \partial_\nu (\rho) \quad (100)$$

Here a shorthand has been used to write $|\psi|^2 = \rho$, but it should be remembered that this quantity exists as a point within the distribution of the particle, not merely as a density.

10.3.3 System Compatibility

All Lagrangian monomials up to dimension 4 have now been collected. That does not always mean these terms form a physically consistent coupled system. How to determine the κ coupling term, g_1 , g_2 , and g_3 remains unknown. Full specification of the system is necessary before its use. The Maxwell equation has an easily derived conserved current thanks to the construction of its anti-symmetric field tensor:

$$F^{\mu\nu} + F^{\nu\mu} = 0 \quad (101)$$

Taking derivatives $\partial_\mu \partial_\nu$ of $F^{\mu\nu}$ and exploiting the commutativity of differentiation:

$$\partial_\mu \partial_\nu F^{\mu\nu} = \frac{1}{2} (\partial_\mu \partial_\nu F^{\mu\nu} + \partial_\mu \partial_\nu F^{\mu\nu}) = \frac{1}{2} \partial_\mu \partial_\nu (F^{\mu\nu} + F^{\nu\mu}) = 0 \quad (102)$$

Differentiating (100) generates the continuity equation:

$$g_1 \partial_\mu j^\mu + 2g_2 \partial_\mu (A^\mu \rho) + g_3 \square \rho = 0 \quad (103)$$

This represents a Maxwell current of:

$$J_M^\mu = \kappa (g_1 j^\mu + 2g_2 A^\mu \rho + g_3 \partial^\mu \rho) \quad (104)$$

³⁶After a long and fun calculation, raising and lowering indices, renaming dummy indices, and shuffling Kronecker deltas around.

The continuity equation is not a new constraint but one already satisfied by the Maxwell equation. It alone cannot enforce the complete Maxwell equation. As a condition, it is necessary but insufficient. Since the governing equation for ψ has been coupled, it must *also* conserve that current; otherwise both systems are inconsistent or overdetermined. Every monomial that could be added up to dimension 4 has been included. Is there a similar and necessary continuity equation for ψ ? Indeed, to find such a conserved current of the coupled system, the starting phase θ_0 choice symmetry must be put to good use. Since a deviation of θ_0 does not change the physics and the Lagrangian is already free of such terms, there will be an associated conserved current by Noether's theorem. It is possible that it would be identical to the Maxwell current, in which case both terms would have to be kept and uniqueness checked, but proceeding will reveal the answer.

To begin, vary θ by a constant amount and calculate how the differential form of the system shifts, then exploit its invariance. Taking $\theta_0 = \alpha$ phase-shifts the wavefunction: $\psi \rightarrow e^{i\alpha}\psi$, $\psi^* \rightarrow e^{-i\alpha}\psi^*$. Trivial verification reveals this leaves $\mathcal{L}_{\text{total}}$ the same, i.e. $\mathcal{L}_{\text{total}} \rightarrow \mathcal{L}_{\text{total}}$, yielding the following variations for small α :

$$\delta\psi = (e^{i\alpha} - 1)\psi = i\alpha\psi + \mathcal{O}(\alpha^2), \quad \delta\psi^* = (e^{-i\alpha} - 1)\psi^* = -i\alpha\psi^* + \mathcal{O}(\alpha^2), \quad (105)$$

$$\delta(\partial^\mu\psi) = \partial^\mu(\delta\psi) = i\alpha\partial^\mu\psi + \mathcal{O}(\alpha^2), \quad \delta(\partial^\mu\psi^*) = \partial^\mu(\delta\psi^*) = -i\alpha\partial^\mu\psi^* + \mathcal{O}(\alpha^2). \quad (106)$$

Now, expand $\delta\mathcal{L}_{\text{total}}$ with the chain rule, noting that it vanishes through the identified symmetry:

$$0 = \delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\psi}\delta\psi + \frac{\partial\mathcal{L}}{\partial\psi^*}\delta\psi^* + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\delta(\partial_\mu\psi) + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi^*)}\delta(\partial_\mu\psi^*) \quad (107)$$

To keep the notation tractable, two momenta are defined:

$$\Pi^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}, \quad \Pi^{*\mu} = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi^*)} \quad (108)$$

Since the partial derivative commutes with the variation of ψ , the product rule extracts another element in the usual way; this element is then grouped with the non-derivative variation:

$$\Pi^\mu\partial_\mu(\delta\psi) = \partial_\mu(\Pi^\mu\delta\psi) - \partial_\mu(\Pi^\mu)\delta\psi \quad (109)$$

Substituting the same relation for the conjugate,

$$\delta\mathcal{L} = \left(\frac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu(\Pi^\mu)\right)\delta\psi + \left(\frac{\partial\mathcal{L}}{\partial\psi^*} - \partial_\mu(\Pi^{*\mu})\right)\delta\psi^* + \partial_\mu(\Pi^\mu\delta\psi + \Pi^{*\mu}\delta\psi^*) \quad (110)$$

By construction, the Euler-Lagrange equation eliminates the first two terms. Since $\delta\mathcal{L}$ vanishes, the remaining term gives the conserved current continuity equation. Now, substitute the variation and drop α to define:

$$\partial_\mu(\Pi^\mu\delta\psi + \Pi^{*\mu}\delta\psi^*) = 0 \Rightarrow \boxed{j_N^\mu = i(\Pi^\mu\psi - \Pi^{*\mu}\psi^*)} \quad (111)$$

Keep in mind that the **total** Lagrangian momenta in (108) must be taken. A substantial amount of the usual raising and lowering of indices yields:

$$j_N^\mu = i(\psi \partial_\mu \psi^* - \psi^* \partial_\mu \psi) + g_1 A^\mu \rho \quad (112)$$

This current is not normalized because of the unitless definition of ψ and the arbitrary choice of α which was divided away. Notice how this resembles the Maxwell current, in that it contains two terms corresponding to the monomials, but is missing the g_3 term. Since the current is conserved, its partial derivative is zero, so multiplying (112) through by $\frac{g_1}{2}$ yields:

$$\frac{g_1}{2} j_N^\mu = g_1 \frac{i}{2} (\psi \partial_\mu \psi^* - \psi^* \partial_\mu \psi) + \frac{1}{2} g_1^2 A^\mu \rho = g_1 j^\mu + \frac{1}{2} g_1^2 A^\mu \rho \quad (113)$$

The conservation of this current implies:

$$\partial_\mu \left(\frac{g_1}{2} j_N^\mu \right) = g_1 \partial_\mu j^\mu + \frac{1}{2} g_1^2 \partial_\mu (A^\mu \rho) = 0 \Rightarrow \boxed{g_1 \partial_\mu j^\mu = -\frac{1}{2} g_1^2 \partial_\mu (A^\mu \rho)} \quad (114)$$

This differs significantly from the Maxwell continuity equation (103), which indicates the pair of coupled systems is potentially *not compatible*³⁷ except for certain situations where both continuity equations can be fulfilled. The continuity equation (103) is compatible with (or contained in) the Maxwell equation, while the continuity equation (114) is contained in the wave equation being built. Both partial differential equations must be satisfied or they will be inconsistent and will diverge in their overall configuration space. Only in some situations will there be general compatibility. In differential geometry terminology this scenario entails an **overdetermined system**, as the Maxwell equation would not automatically satisfy the continuity equation of the wave equation, nor would the wave equation satisfy the continuity equation of the Maxwell equation. Both equations need to be satisfied simultaneously, typically restricting the configuration space to trivial solutions, most likely 0 everywhere.

Compatibility demands that the continuity equation of the Maxwell PDE be consistent with the continuity equation for the wavefunction PDE, and vice versa. Every term in either continuity equation is a linearly independent operator, meaning the null space of both needs to be considered and the appropriate physically realistic constraint determined. Rather than working through the Frobenius theorem, simple algebra and physical reasoning arrive at the same conclusion.

Substitution of the ψ continuity equation (114) into the Maxwell continuity equation (103), followed by rearrangement, yields:

$$(2g_2 - \frac{1}{2} g_1^2) \partial_\nu (A^\nu \rho) + g_3 \square \rho = 0 \quad (115)$$

The two differential operators are linearly independent, which implies that they need to be satisfied individually. The most severe is g_3 : in situations where there is no field in some compact support $A^\nu = 0$ all through to the boundary, if $g_3 \neq 0$:

$$\square \rho = 0 \quad (116)$$

This admits only harmonic solutions for density. In any steady state, this condition cannot localize a particle as it reduces to the Laplacian $\delta \rho = 0$, and since the boundary

³⁷A term from differential geometry; see the Frobenius theorem for further details.

conditions in the compact support are zero, ρ is 0 everywhere. Only trivial solutions are admitted as this implies $\psi = 0$; therefore,

$$\boxed{g_3 = 0} \quad (117)$$

The result follows naturally, since a harmonic in a continuity equation creates severe restrictions on dynamics. Even so, the g_3 term remains a smaller correction since it depends on the gradient across the already finer ψ —itself coarse-grained under macroscopic and typical laboratory conditions (i.e. non-superconducting temperatures). That result is nevertheless examined shortly.

This leaves the following condition:

$$(2g_2 - \frac{1}{2}g_1^2)\partial_\nu(A^\nu\rho) = 0 \quad (118)$$

Assume $g_2 \neq \frac{1}{4}g_1^2$; this implies $\partial_\nu(A^\nu\rho) = 0$ for any vector potential A^ν and any ρ . Now, A^ν contains a gauge choice, and the equation should be compatible with any gauge. Since the term is manifestly not gauge-invariant, it too must be eliminated. The same requirement follows from an alternative argument. Suppose a gauge other than the Lorenz gauge is chosen, i.e. $\partial_\nu A^\nu \neq 0$. Expanding via the product rule:

$$\partial_\nu(A^\nu\rho) = (\partial_\nu A^\nu)\rho + A^\nu\partial_\nu\rho = 0 \quad (119)$$

The previous partial differential equation may be rearranged into separate solution spaces:

$$\frac{\partial_\nu A^\nu}{A^\nu} = -\frac{\partial_\nu\rho}{\rho} = \lambda \quad (120)$$

This makes the density, a gauge-invariant ontic quantity, dependent on gauge—a contradiction. It restricts the functional form of the vector potential, which is not only nonphysical but restricts even the configuration space in addition:

$$\boxed{g_2 = \frac{1}{4}g_1^2, \quad g_3 = 0.} \quad (121)$$

A significant result emerges that is identical to what arises from the postulation of “local U(1) symmetry,” yet only the machinery of integrability conditions from the 1800s was required to arrive there without invoking unjustified postulates.

All the incompatible, redundant, and non-physical 4th-dimensional coupling terms have been eliminated. The task is not quite complete, however: for this linearized coupling to be unique, the analysis cannot simply stop at dimension 4 but must continue to dimension ∞ , because the dimensional argument was purely heuristic.

10.3.4 Intermission: Maximal Coupling

What if one attempts to add further terms? Seeing g_3 eliminated by incompatibility may prove discouraging, but at least examining what is left behind by stopping there is warranted for a uniqueness claim on the Klein–Gordon derivation. Those hoping to see additional physics will be disappointed to learn that due to parity restrictions, no dimension-5 monomials meet the selection rules determined to be necessary so far.

Dimension 6 is interesting. It contains a rich number of terms, such as $A_\mu A^\mu \partial_\nu \psi \partial_\nu \psi^*$, which remain with the target selection rules but fail the integrability condition between

the systems. Repeating the procedure by hand is impractical, so software was developed to investigate the forms. Overall, 7 monomials survive the selection rules and combine into 4 terms, with 3 new independent parameters whose values must be determined experimentally. Without exception, every one of these terms contains a gradient in one field or another, rendering them 2nd-order (in energy) corrections to an already coarse governing equation. Since no account of spin has been developed, these terms may safely be taken to vanish if a strong experimental match with the discovered form can be demonstrated. The spin corrections and soliton-soliton interaction would dominate any such terms.

From dimension 7 onward, there are only monomials that leave behind nonlinear terms in one of the PDEs, so the search ends there. By dimensional analysis, these terms are all going to be either zero or so exceedingly small that they are immeasurable and likely a poor representation of what is being modeled, due to the approximation committed in the search for local bilinear interaction.

10.4 Derivation of the Klein–Gordon Equation

Due to the way the coupling constant structure was strategically selected, the ψ domain can be completely specified with just one coupling constant: g_1 . First, the combined Lagrangian is varied to produce the coupled equation of motion. The $\mathcal{L}_f$ term does not participate in the variation, and so the κ coefficient can be discarded.

Without further delay, varying ψ^* produces the coupled wavefunction:

$$\frac{\partial \mathcal{L}}{\partial \psi^*} = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi^*)} \right) \quad (122)$$

The left-hand side is simply:

$$\frac{\partial \mathcal{L}}{\partial \psi^*} = -\frac{m^2 c^2}{\chi^2} \psi + g_1 \frac{i}{2} A^\mu \partial_\mu \psi + \frac{1}{4} g_1^2 A_\mu A^\mu \psi \quad (123)$$

To find the inner part of the right-hand side, apply the metric $g^{\alpha\mu}$ to the appropriate coefficient:

$$\frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi^*)} = g^{\alpha\mu} \partial_\alpha \psi - \frac{i}{2} g_1 A^\mu \psi = \partial^\mu \psi - \frac{i}{2} g_1 A^\mu \psi \quad (124)$$

Differentiating the previous term:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi^*)} \right) = \partial_\mu \partial^\mu \psi - \frac{i}{2} g_1 A^\mu \partial_\mu \psi - \frac{i}{2} g_1 \psi \partial_\mu A^\mu \quad (125)$$

Expanding this yields the complete equation:

$$-\frac{m^2 c^2}{\chi^2} \psi + g_1 \frac{i}{2} A^\mu \partial_\mu \psi + \frac{1}{4} g_1^2 A_\mu A^\mu \psi - \partial_\mu \partial^\mu \psi + \frac{i}{2} g_1 A^\mu \partial_\mu \psi + \frac{i}{2} g_1 \psi \partial_\mu A^\mu = 0 \quad (126)$$

Combining the common terms and ensuring the leading derivatives are positive:

$$\partial_\mu \partial^\mu \psi + \frac{m^2 c^2}{\chi^2} \psi - i g_1 A^\mu \partial_\mu \psi - \frac{g_1^2}{4} A_\mu A^\mu \psi - \frac{i g_1}{2} \psi \partial_\mu A^\mu = 0 \quad (127)$$

Later in the paper, $\chi = \hbar$ and $g_1 = \frac{2q}{\hbar}$ are determined experimentally... but the temptation to substitute them at this early stage proves irresistible, and so they are used here for *full effect*:

$$\boxed{\partial_\mu \partial^\mu \psi + \frac{m^2 c^2}{\hbar^2} \psi - \frac{2qi}{\hbar} A^\mu \partial_\mu \psi - \frac{q^2}{\hbar^2} A_\mu A^\mu \psi - \frac{iq}{\hbar} \psi \partial_\mu A^\mu = 0} \quad (128)$$

Although it is not yet obvious, what was deemed impossible for a century has now been done. The Klein–Gordon equation has been obtained, the non-relativistic limit being the Schrödinger equation. This very point was reached with zero postulation, only deductive reasoning through the process of elimination and adherence to stringently established Lorentz invariance. Other than the covariant forms, the mathematics used is from the Victorian era. The ideas are from the era of Lord Kelvin—essentially from first principles. $\square$

To compact the equation, it is now factorized. This will help derive the Schrödinger equation in the next subsection. First, recall the following identity from the product rule:

$$\partial_\mu (A^\mu \psi) = \psi \partial_\mu A^\mu + A^\mu \partial_\mu \psi \quad (129)$$

Then split the $\frac{2qi}{\hbar}$ coefficient into two, pair the coefficient with the divergence term, and go through further steps:

$$\partial_\mu \partial^\mu \psi + \frac{m^2 c^2}{\hbar^2} \psi - \frac{iq}{\hbar} A^\mu \partial_\mu \psi - \frac{q^2}{\hbar^2} A_\mu A^\mu \psi - \left(\frac{iq}{\hbar} A^\mu \partial_\mu \psi + \frac{iq}{\hbar} \psi \partial_\mu A^\mu \right) = 0 \quad (130)$$

$$\partial_\mu \partial^\mu \psi - \frac{iq}{\hbar} \partial_\mu (A^\mu \psi) - \frac{iq}{\hbar} A^\mu \partial_\mu \psi - \frac{q^2}{\hbar^2} A_\mu A^\mu \psi + \frac{m^2 c^2}{\hbar^2} \psi = 0 \quad (131)$$

$$\partial_\mu \left(\partial^\mu \psi - \frac{iq}{\hbar} A^\mu \psi \right) - \frac{iq}{\hbar} A_\mu \left(\partial^\mu \psi - \frac{iq}{\hbar} A^\mu \psi \right) + \frac{m^2 c^2}{\hbar^2} \psi = 0 \quad (132)$$

Define the common inner term as $D^\mu \equiv \partial^\mu - \frac{iq}{\hbar} A^\mu$:

$$\partial_\mu D^\mu \psi - \frac{iq}{\hbar} A_\mu D^\mu \psi + \frac{m^2 c^2}{\hbar^2} \psi = \left(\partial_\mu - \frac{iq}{\hbar} A_\mu \right) D^\mu \psi + \frac{m^2 c^2}{\hbar^2} \psi = 0 \quad (133)$$

Recognizing the covariant form of D_μ , one obtains the familiar and popular form of the Klein–Gordon equation:

$$\boxed{D_\mu D^\mu \psi + \frac{m^2 c^2}{\hbar^2} \psi = 0} \quad (134)$$

No “minimal coupling” ansatz was needed here; the result was reached through pure deduction. To leave the coupling constant undetermined, redefine D^μ as:

$$\boxed{D^\mu \equiv \partial^\mu - i \frac{g_1}{2} A^\mu} \quad (135)$$

For now, χ is substituted back into the equation in place of $\hbar$ until experimental determination in the next chapter, essentially reverting to (128).

10.5 Schrödinger Equation Derivation

At long last, the derivation most readers have been anticipating: deriving the Schrödinger equation itself! At this point, the result is a fait accompli, but the path to it is illuminating because it assumes a rest frequency for the electron even in the standard formulation. In contrast, this was determined as the earliest deduction, and it has been the Archimedean pivot enabling derivation of everything so far. A practical need also exists: the Klein–Gordon equation predicts the wrong kind of fine structure for hydrogen! The Schrödinger equation is needed to recover χ properly. To that end, usage reverts to g_1 and χ except when the final result is presented.

The Schrödinger equation is the non-relativistic limit of the Klein–Gordon equation, but the path to it proves most illuminating. Returning to the energy-frequency relation and taking a limit $v \ll c$:

$$E = \chi\omega = \gamma\chi\omega_0 = \frac{\chi\omega_0}{\sqrt{1-v^2/c^2}} \approx \chi\omega_0 + \frac{1}{2}\chi\omega_0 \frac{v^2}{c^2} = E_0 + \frac{E}{2c^2}v^2 \quad (136)$$

Since $E = mc^2$:

$$E \approx \chi\omega_0 + \frac{1}{2}mv^2 \quad (137)$$

So the energy is the translation of the soliton’s rest frequency into energy, plus its kinetic energy. That indicates the kinetic energy resides entirely in the Lorentz-boosted excitation of the soliton’s rest energy, akin to an extended-body Doppler shift. Factoring that from the wavefunction’s θ representation yields this kinetic modulation wave, carried by the extremely high-frequency rest soliton.

$$\psi(\mathbf{x}, t) = \Psi(\mathbf{x}, t)e^{-i\omega_0 t} \quad (138)$$

This resembles demodulation, but here the representation contains only one carrier wave, not two sidebands³⁸. The complete magnetic field and potential coupled Schrödinger equation could be recovered, but only the electrostatic one is needed for present purposes—the one most people are familiar with. So to simplify matters, take $A^\mu = (\phi/c, \mathbf{0})$. In this case, D^μ simplifies to:

$$D^\mu = \left(\partial^t - i\frac{g_1\phi}{2c}, \nabla \right) \quad (139)$$

Taking the self-product and applying the commutative property of differential operators, expanding the Lorentz-invariant differential operator $\partial_\mu = (\frac{1}{c}\partial_t, \nabla)$:

$$D_\mu D^\mu = \frac{1}{c^2} \left(\partial_t - i\frac{g_1\phi}{2} \right)^2 - \nabla^2 \quad (140)$$

With this now derived, the electrostatic-only Klein–Gordon equation (134) is:

$$\frac{1}{c^2} \left(\frac{\partial}{\partial t} - i\frac{g_1\phi}{2} \right)^2 \psi - \nabla^2 \psi + \frac{m^2 c^2}{\chi^2} \psi = 0 \quad (141)$$

Substitute (138):

³⁸The spurious negative “energy” solutions of the Klein–Gordon equation are ignored. Interpreting them as negative “frequency” arising from the Fourier transform process that yielded the dispersion relation is an obvious route.

$$\frac{1}{c^2} \left(\frac{\partial}{\partial t} - i \frac{g_1 \phi}{2} \right)^2 \Psi e^{-i\omega_0 t} - \nabla^2 (\Psi e^{-i\omega_0 t}) + \frac{m^2 c^2}{\chi^2} \Psi e^{-i\omega_0 t} = 0 \quad (142)$$

Multiply both sides by $e^{i\omega_0 t}$ and expand the first term:

$$e^{i\omega_0 t} \frac{1}{c^2} \left(\frac{\partial}{\partial t} - i \frac{g_1 \phi}{2} \right)^2 \Psi e^{-i\omega_0 t} - \nabla^2 \Psi + \frac{m^2 c^2}{\chi^2} \Psi = 0 \quad (143)$$

$$e^{i\omega_0 t} \frac{1}{c^2} \left(\frac{\partial^2}{\partial t^2} - i g_1 \phi \frac{\partial}{\partial t} - \frac{g_1^2 \phi^2}{4} \right) \Psi e^{-i\omega_0 t} - \nabla^2 \Psi + \frac{m^2 c^2}{\chi^2} \Psi = 0 \quad (144)$$

The first derivative is:

$$\frac{\partial}{\partial t} (\Psi e^{-i\omega_0 t}) = \frac{\partial \Psi}{\partial t} e^{-i\omega_0 t} - i\omega_0 \Psi e^{-i\omega_0 t} = \left(\frac{\partial \Psi}{\partial t} - i\omega_0 \Psi \right) e^{-i\omega_0 t} \quad (145)$$

And then the second derivative:

$$\frac{\partial^2}{\partial t^2} (\Psi e^{-i\omega_0 t}) = \left(\frac{\partial^2 \Psi}{\partial t^2} - 2i\omega_0 \frac{\partial \Psi}{\partial t} - \omega_0^2 \Psi \right) e^{-i\omega_0 t} \quad (146)$$

Apply the operation on the first term through the product rule and group the terms:

$$\left(\frac{\partial^2}{\partial t^2} - i g_1 \phi \frac{\partial}{\partial t} - \frac{g_1^2 \phi^2}{4} \right) \Psi e^{-i\omega_0 t} = \left(\frac{\partial^2 \Psi}{\partial t^2} - i(2\omega_0 + g_1 \phi) \frac{\partial \Psi}{\partial t} - (\omega_0 - \frac{g_1 \phi}{2})^2 \Psi \right) e^{-i\omega_0 t} \quad (147)$$

Now the demodulation term in the equation can finally be removed. Substitute this result into (144):

$$\frac{1}{c^2} \left(\frac{\partial^2 \Psi}{\partial t^2} - i(2\omega_0 + g_1 \phi) \frac{\partial \Psi}{\partial t} - (\omega_0 - \frac{g_1 \phi}{2})^2 \Psi \right) - \nabla^2 \Psi + \frac{m^2 c^2}{\chi^2} \Psi = 0 \quad (148)$$

Combine the Ψ terms by substituting $E = mc^2$ and $E = \chi\omega_0$:

$$E^2 = (mc^2)^2 = \chi^2 \omega_0^2 \Rightarrow \frac{m^2 c^2}{\chi^2} = \frac{\omega_0^2}{c^2} \quad (149)$$

Substituting this into (148) removes the ω_0^2 term:

$$\frac{1}{c^2} \left(\frac{\partial^2 \Psi}{\partial t^2} - i(2\omega_0 + g_1 \phi) \frac{\partial \Psi}{\partial t} + \omega_0 g_1 \phi \Psi - \frac{g_1^2 \phi^2}{4} \Psi \right) - \nabla^2 \Psi = 0 \quad (150)$$

Multiplying by $\frac{\chi}{2m}$ transforms the first coefficient's denominator into energy, which is $\chi\omega_0$, hence:

$$\frac{1}{2\omega_0} \left(\frac{\partial^2 \Psi}{\partial t^2} - i(2\omega_0 + g_1 \phi) \frac{\partial \Psi}{\partial t} + \omega_0 g_1 \phi \Psi - \frac{g_1^2 \phi^2}{4} \Psi \right) - \frac{\chi}{2m} \nabla^2 \Psi = 0 \quad (151)$$

The rest frequency of the electron soliton ω_0 is unimaginably high, around 10^{20} Hz. For non-relativistic setups with relatively moderate potentials ϕ , the terms that lack a ω_0 prefactor are eliminated³⁹:

³⁹Dimensional analysis justifies this especially for the second-order derivative in time.

$$\frac{1}{2\omega_0} \left(-i(2\omega_0) \frac{\partial \Psi}{\partial t} + \omega_0 g_1 \phi \Psi \right) - \frac{\chi}{2m} \nabla^2 \Psi + \mathcal{O}(\omega_0^{-1}) = 0 \quad (152)$$

Which reduces to:

$$-i \frac{\partial \Psi}{\partial t} + \frac{1}{2} g_1 \phi \Psi - \frac{\chi}{2m} \nabla^2 \Psi = 0 \quad (153)$$

Multiplying by χ :

$$-i\chi \frac{\partial \Psi}{\partial t} + \frac{\chi}{2} g_1 \phi \Psi - \frac{\chi^2}{2m} \nabla^2 \Psi = 0 \quad (154)$$

The following section will establish that $\chi = \hbar$ and $g_1 = \frac{2q}{\chi}$. The electrostatic potential is defined as $V = q\phi$, so rearranging and substituting yields the time-dependent Schrödinger equation:

$$\boxed{-\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi = i\hbar \frac{\partial \Psi}{\partial t}} \quad (155)$$

Normally, to obtain this result from the Klein–Gordon equation, physicists have to attribute to the electron a rest frequency. This was not even assumed or postulated here but deduced through a long process of elimination, and in the end the Schrödinger equation dropped out with all of its associated machinery.

For present purposes, bring back the to-be-determined constants:

$$-\frac{\chi^2}{2m} \nabla^2 \Psi + \frac{1}{2} g_1 \chi \phi \Psi = i\chi \frac{\partial \Psi}{\partial t} \quad (156)$$

10.6 Bilinear Property

Looking back at the source-free equation in (62), a huge effort went into adding just one extra term. To rearrange the equation and improve clarity, define the following operator:

$$L_S \equiv i \frac{\partial}{\partial t} + \frac{\chi}{2m} \nabla^2 \quad (157)$$

In the source-free case the soliton's wave equation becomes:

$$L_S \Psi = 0 \quad (158)$$

To couple this wave equation with a space-varying electrostatic source, multiply the wavefunction by the potential along with the coupling constant:

$$L_S \Psi = \frac{1}{2} g_1 \phi \Psi \quad (159)$$

The lowest-order, low-energy, and non-relativistic limit of the relation was sought, and it ended up being... the lowest-order term possible! Can further terms be added? Alas, no. The next-order term vanishes under the non-relativistic approximation (i.e., the g_2 or g_1^2 term as it eventually appears). The Schrödinger equation emerges as the unique low-energy non-relativistic approximation of the nonlinear field interaction (75) with a

spinless charged particle.⁴⁰ From here, superpositions of solutions, eigenfunctions, and all the other machinery follow, justified in an ontological manner, not a Hilbert formalism.

More significantly, since the starting point was the electron as a field excitation (proving the main thrust of Poincaré’s “end of matter” paper through deductive reasoning, then carrying forward that realization to this point), it should be observed that applying an electrostatic voltage to the electron results in multiplication of fields only.

That should make it clear why multi-body interaction is another class of problem and the bilinear property fails. This outcome is expected because only $L_i[F^{\mu\nu} \otimes S_e[\psi]]$ was modeled, leaving out the higher-order terms and soliton-soliton interaction. How does modern computational quantum chemistry handle this? Through approximations whose limitations are quite suggestive. Density Functional Theory (DFT) reduces the intractable many-body problem to a tractable one-electron problem, but not analytically: the exchange-correlation functional that encapsulates quantum many-electron interactions is unknown. Most methods use experimental data to drive the computation. The higher-order terms reside precisely here, and any precise soliton-soliton interaction dynamic would have to reproduce these empirical values computationally.

This situation reveals something even deeper about what has been derived. Even when solved exactly, the Schrödinger equation describes only the linear, low-energy, bilinear-coupled regime. It cannot deal with the soliton passing through the nucleus as the soliton does in the $2s$ subshell, where the detectable Lamb shift occurs. Being linear, the Schrödinger equation cannot determine transitional effects without an additional framework (Fermi’s golden rule), which itself approximates the nonlinear dynamics explicitly excluded. One wonders whether modern QED interactions applied to the same problem would recover the bond lengths. If not, then the nonlinear (and nonlocal) field equation with an exact representation of the soliton’s field configuration becomes necessary to address this shortcoming.

⁴⁰The Pauli equation would of course cover spin.

11 Experimental Analysis and Applications

The following are self-contained experimental analyses of foundational experiments. Each was selected to recover one of the χ , g_1 , or κ terms. Each has been selected because it is interesting and reflects the background section, but now with the insight of soliton mechanics. Some are self-contained; some can rely on conclusions resolved in other sub-sections. Each was selected to provide a key insight and remove many of the illusions and outright misinformation imposed by modern pedagogy. This will involve discussing the detailed history of the experiments and the discussions surrounding them, placing the experiments in proper context. These experiments were already summarized in the background section to prepare the reader for this deeper, theoretical and experimental dive.

The section begins by probing experiments with surprising conclusions that physicists felt required revolutionary ideas, then dives into the atomic spectrum, the photoelectric effect, then concludes with the Thomson scattering effect and Compton scattering. The latter will be of utmost importance to the findings of this paper.

11.1 Blackbody Radiation

A classical problem posed by Gustav Kirchhoff is the explanation of the blackbody radiation spectrum. Qualitatively, an ideal blackbody appears black at room temperatures yet in fact emits radiation at all wavelengths, with peak spectral radiance (an expression of power per area per wavelength) at a wavelength that decreases as temperature increases. That trend appears in fig. 3.

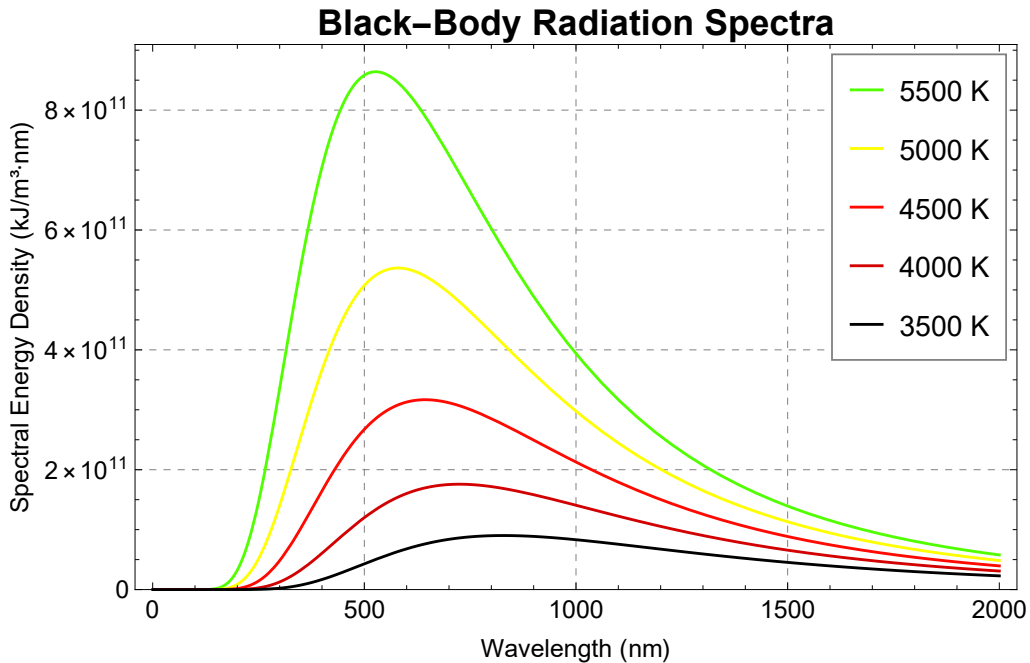


Figure 3: Blackbody radiation spectra for temperatures ranging from 3500 K to 5500 K, showing spectral energy density as a function of wavelength.

The first person to conclusively solve part of this problem was Wilhelm Wien. Wien in fact discovered two interesting findings. In Wien's deep analysis, a famous result from classical mechanics now called 'adiabatic invariance' was used. A system in oscillatory

motion, parameterized by some parameter λ that is slowly varied, has an invariance $I = \frac{1}{2\pi} \oint pdq$ such that:

$$E = I\omega \quad (160)$$

This linearity remains provided that the system stays in the mode and λ is varied only slightly [46]. This is observable when shortening or lengthening the length of a pendulum, for example. The invariance is very similar to χ , but is not intrinsic to the system: without further restrictions, every single mode of frequency ω_n will have its own potentially distinct I_n . Yet, Wien used this theorem in 1893 to find the peak wavelength of an ideal blackbody by considering the volume of the blackbody expanding or contracting as the blackbody remained in equilibrium; Wien then considered the Doppler effect to find a connection to wavelength.

The outcome of Wien's work was twofold. The adiabatic invariance imposed the following restriction on the spectral energy density $B(\nu)$ where $\nu = \omega/2\pi$ is the ordinary frequency:

$$B(\nu, T) = \nu^3 F(\nu/T) \quad (161)$$

By differentiating $B(\nu, T)$ with respect to ν , Wien discovered that the density had a single peak frequency per temperature as observed in experimental data:

$$\lambda_{\text{peak}} = b/T \quad (162)$$

All that remained was for someone to find b and F . Interestingly, the formula is somewhat equivalent to Planck's energy law. Applying the equipartition theorem gives a relationship between energy and temperature: $\langle E \rangle = \frac{n}{2} k_B T$ where k_B is Boltzmann's constant⁴¹. Setting up a proportionality constant b' between the peak energy and the resonator energy, one finds that the peak wavelength formula and equipartition yield:

$$E = b'\nu \quad (163)$$

The hint was there, and Planck did not ignore it. In Planck's world-changing derivation, an ensemble of identical resonators within the blackbody was posited. Planck then considered the entire system in resonance, and also considered the "walls" of the blackbody resonator to be perfectly reflective on the interior, meaning the walls would form nodes in any resonance mode. A useful analogy is a string in one dimension. The string can only resonate in harmonics of its length, characterised by a harmonic number n . These are the permitted modes within the system viewed as a function of its dimensions. To make life easier, Planck used a cube approximation with lengths L without loss of generality that sets up a fundamental frequency f_0 . At the boundary of the system (and because of the mirror-like walls), there is this restriction on modes⁴²:

$$\nu_n = n \cdot f_0 = n \cdot \frac{c}{2L} \quad (164)$$

So far nothing too wild had been introduced by Planck. The key step was this: each resonator could only hold a fixed and specific energy for every individual mode it

⁴¹Ironically, Boltzmann's constant was defined by Planck alongside the Planck constant. And yet, the constant is etched on Boltzmann's tombstone in Vienna...

⁴²Planck describes this differently in the paper, focusing on the sum of energies thus frequencies, but the description is a conclusion drawn from the same setup.

contributes to, and each resonator could hold many modes in superposition but each mode only contained this amount of energy. In one dimension the expression reduces to:

$$\boxed{E_n = nhf_0 = nE_0} \quad (165)$$

and consequently identified the function $F(\nu/T)$, producing what is known as Planck's distribution law:

$$B(\nu, T) = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{e^{h\nu/k_B T} - 1} \quad (166)$$

Why did Planck introduce the fixed energy per mode restriction? Planck's partition function, derived from entropy considerations, revealed a problem: if a resonator could carry an arbitrary amount of energy per mode, given the mode count increases with the square of frequency⁴³, the equipartition theorem would dictate that the energy distribution tends towards infinity as frequency increases. That result is attributed to Rayleigh, a result that was only arrived at *after* Planck's distribution—despite what today's pedagogy might suggest. Thankfully, reality works the way Planck deduced, or lumps of coal would be dangerous to keep around.

Planck's original derivation appeared in a paper titled “On the Theory of the Law of Energy Distribution in the Normal Spectrum” [47]. The “quanta” idea came later, because of Einstein, who decided to postulate that the quantization was happening to the field itself. This was widely rejected when first proposed, and as the work here demonstrates, physicists, including Planck himself, were **correct to reject Einstein's idea**. The “quantization” was simply the discrete nature of the electrons participating in the field resonance condition of the blackbody radiation. It was indeed exactly as Planck had said, and Planck had derived the non-relativistic limit of the electron soliton formula of section 7, in the same way the Schrödinger equation was derived here.

Planck's mode restriction is an unnecessary postulation in retrospect, since the restriction follows directly from $E = \chi\omega$ —the soliton can only store that increment of energy for the kinetic energy it absorbs from the field resonance condition. The electron is a field excitation and in a sense the entire blackbody is as well, and the complete system and subsystem are in a harmonic resonance condition when emitting radiation as all experimenters observe.

Ironically, this was the accepted view during Planck's time, but the view was gradually replaced with what became the photon idea. Historically, the electron was a newly discovered particle, and it would be 10 years before Robert Millikan would confirm the quantization of electric charge—a concept not many had accepted. Perhaps if history had been ordered differently, Planck's hypothetical resonator would have had an obvious candidate? The result has now been determined. The value of χ is the Planck constant h divided by 2π :

$$\boxed{\chi = \frac{h}{2\pi} = \hbar} \quad (167)$$

Measuring the blackbody radiation yields the constant.

⁴³Picture one octant of a 3D axis as a sphere expands. Each shell captures modes at a rate proportional to the surface area of the shell, hence the square dependence.

11.2 Room-Temperature and Superconducting Electromagnetics

One nice outcome would be to analyze some experimental observations that will set the value of κ and g_1 and completely determine the coupling between the Maxwell field and the wavefunction representation. Strictly speaking, this proves **unnecessary** for the final result: χ can be discovered experimentally, and from there only the g_1 coupling term remains to be determined as the κ term cancels out in the ψ -sector Lagrangian variation. Even so, fixing the terms by applying macroscopic constraints in the field sector is a powerful demonstration of the consistency and insight the method offers. This is the sole route to κ . It will demonstrate the consistency of the method across all domains, and that what has been derived in the coupled system is a true description of both physical systems.

Plugging (121) into (100) yields a unique but undetermined-parameter Maxwell equation:

$$\frac{1}{\mu_0} \partial_\mu F^{\mu\nu} = \kappa g_1 j^\nu + \frac{\kappa}{2} g_1^2 A^\nu \rho = \kappa g_1 j^\nu + \frac{\kappa}{2} g_1^2 A^\nu |\psi|^2 \quad (168)$$

For a non-relativistic experimental setup, it is helpful to express this equation in vector form, with $A_\mu = (\phi/c, \mathbf{A})$, thus $A^\mu = (\phi/c, -\mathbf{A})$. Beginning with Coulomb's law (Gauss's law):

$$\partial_i F^{i0} = \frac{1}{c} \nabla \cdot \mathbf{E} \Rightarrow \boxed{\nabla \cdot \mathbf{E} = \kappa \mu_0 g_1 c j^0 + \kappa \mu_0 \frac{g_1^2}{2} \phi |\psi|^2} \quad (169)$$

Here a restriction becomes immediately obvious upon encountering the wavefunction density: only *one* electron and *one* field configuration have been considered. The result has limited utility, however, since field configurations require a charge density and current distribution. To overcome this restriction, the bilinear property and statistical modeling of multi-particle interactions will prove essential.

11.2.1 Room-Temperature Phenomenology

Coulomb's law under room-temperature conditions (without superconduction) has been verified to an extraordinary degree, 10^{16} or higher, and the equation for a total charge distribution ρ_t is:

$$\nabla \cdot \mathbf{E} = \frac{\rho_t}{\varepsilon_0} \quad (170)$$

Replacing the right-hand side numerator with the macroscopic current $J_t^\mu = (c\rho_t, \mathbf{J})$:

$$\nabla \cdot \mathbf{E} = \frac{J_t^0}{\varepsilon_0 c} \quad (171)$$

How to incorporate N solitons into the equation must be considered, along with their mutual interactions, despite not modeling these explicitly. At room temperatures, scattering collisions occur frequently. Solitons may pass over one another, acquiring phase shifts. Higher background fields produce absorption, vibration, and numerous complex modes for each individual soliton. Additionally, θ_0 may follow a random distribution.

Using degrees of freedom for gauge-fixing assumed only one value existed. For multiple particles, that assumption breaks down. Consequently, phase relationships among solitons become completely random. Interference results, along with overall cancellation of $|\psi|^2$, rendering the density indistinguishable from thermal noise.

Conversely, the gradient in phase represents the momentum of each particle in equation (42) which, at least in a free conducting medium without correlations, will be additive. The bilinear property may be used to compute a total current density and therefore a total charge density of:

$$j_t^\mu = \sum_i j_i^\mu = N \mathbb{E} [j^\mu] \quad (172)$$

where $\bar{j}^\mu$ is the average current density. This fixes the normalized ψ -sector dynamics.

Now since the ψ -sector interaction canceled out the wavefunction density, via (169) the macroscopic average electric field divergence is:

$$\mathbb{E} [\nabla \cdot \mathbf{E}] = \kappa \mu_0 g_1 c j_t^0 + N \kappa \mu_0 \frac{g_1^2}{2} \phi \mathbb{E} [\psi^* \psi] = \kappa \mu_0 g_1 c j_t^0 \quad (173)$$

The expected value of the wavefunction density was completely suppressed by room-temperature decoherence. Applying (169) and equating the field:

$$\frac{J_t^0}{\varepsilon_0 c} = \kappa g_1 \mu_0 c j_t^0 \quad (174)$$

Using (42) (i.e., $J_t^\mu = \frac{q\chi}{m} j_t^\mu$) and canceling out a common j_t^0 term:

$$\boxed{\kappa g_1 = \frac{q\chi}{mc^2 \varepsilon_0 \mu_0} = \frac{q\chi}{m}} \quad (175)$$

This yields the first constraint on g_1 . One further constraint will determine both. There is beauty in that the combined coupling terms is a product of all the attributes terms of an electron, charge, frequency and mass. Room-temperature measurements have been exhausted; to proceed further with this method, deep-freeze temperatures are needed!

11.2.2 Cryogenic Superconductors

In 1911, Heike Kamerlingh Onnes of the Netherlands discovered what would later be understood as phase coherence at macroscopic scales. Three years earlier, Onnes reached an incredible milestone in cryogenics, liquefying helium at 4.2 Kelvin. During those early years, there was a great debate about what happened to the conductivity of metals as they were cooled down. Many thought the resistance would fall to zero gradually as absolute zero was approached, but some thought the electrons would “freeze” and stop conducting, meaning the resistance would go up. A scan of the original result is included in this section in figure 4, showing the very abrupt drop in resistance.

Onnes chose mercury as his metal of choice due to the ease of distillation into very high purities⁴⁴ with liquid helium. He then discovered that the conduction abruptly dropped to zero. Onnes had discovered the first superconductor. He later discovered the same phenomenon in tin and lead. Consider what this implies, remembering Ohm’s law with σ being conductivity:

⁴⁴Impurities would spoil conduction measurements.

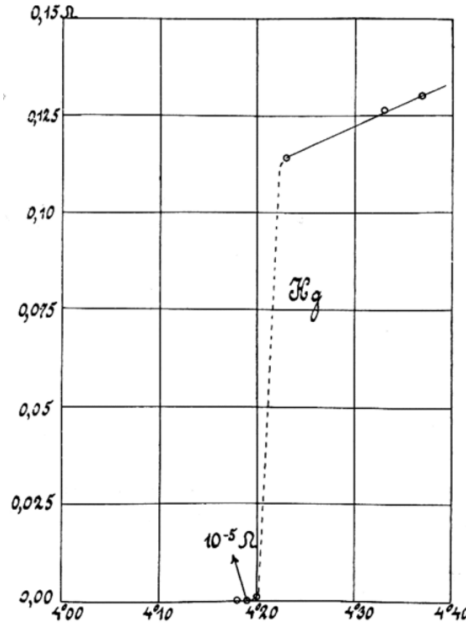


Figure 4: The first observation of superconductivity: electrical resistance of mercury (Hg) as a function of temperature. The resistance drops abruptly from approximately $0.125 \, \Omega$ to below $10^{-5} \, \Omega$ at the critical temperature $T_c \approx 4.2 \, \text{K}$.

$$\mathbf{E} = \frac{1}{\sigma} \mathbf{J} \quad (176)$$

When a superconductor hits its critical temperature, $\frac{1}{\sigma}$ becomes 0. This has been verified by looping the circuit and measuring for decay—no such decay has been observed for years after some starting momentum was injected into the cryogenically cooled conductor. This obviously meant that:

$$\mathbf{E}|_{\text{bulk}} \approx 0 \quad (177)$$

This effect was not well understood and presented another mystery. What happens to the magnetic field at the moment criticality is reached? Does the field “freeze” in the bulk of the conductor? It did not; instead, it was observed to be expelled by the superconductor, meaning $\mathbf{B}|_{\text{bulk}} \approx 0$! Yet, if the electric field was zero, how could that be? A static electric field implies a static magnetic field.

In 1935, two German brothers, Fritz and Heinz London, developed a brilliant phenomenological extension to Maxwell’s equations. That was not explained until 1957 with the combined effort of some of the greatest condensed matter physics groups, eventually leading to the Bardeen-Cooper-Schrieffer theory of superconductors. The popular⁴⁵ website Wikipedia claims that derivation from first principles is considered impossible...

“While it is important to note that the above equations cannot be formally derived”⁴⁶

... Yet, applying nothing but 1800s mathematics and physical principles so far, the London equation has in actuality *already* been derived—it is the g_2 term! All that re-

⁴⁵Though non-ideal as a scholarly source.

⁴⁶https://en.wikipedia.org/w/index.php?title=London_equations&oldid=1327053077

mains is carefully extracting the parameters from the observed phenomena to find κ .⁴⁷ Fortunately, because the treatment so far has been for spinless solitons, a model exists of exactly what electrons become in a superconducting material according to BCS theory: a “Cooper pair” acting as a composite spinless particle. No reliance on such a theory is necessary, however—only fitting a parameter discovered while coupling the Maxwell equation to the dispersion-relation-based wavefunction of an electron soliton.

A closer look at this frozen magnetic field phenomenon comes from extracting Ampère’s law from (168). The prior section considered the timelike dimension; now the same treatment will apply to the spacelike dimension. Starting from a textbook result:

$$\frac{1}{\mu_0} \partial_\mu F^{\mu i} = \frac{1}{\mu_0} (\nabla \times \mathbf{B})_i - \varepsilon_0 \frac{\partial \mathbf{E}_i}{\partial t} \quad (178)$$

In the regime of focus (criticality), $\mathbf{E} = 0$ so the displacement current component should be zero. Therefore, the curl component absorbs the RHS of (168) entirely. Writing this out in vector form while being careful with the signature:

$$\nabla \times \mathbf{B} = \mu_0 \left(\kappa g_1 \mathbf{j} - \frac{\kappa}{2} g_1^2 \mathbf{A} \rho \right) \quad (179)$$

where $\mathbf{j}$ is a vector composed of the spacelike components of j^μ . Now, take another curl of this equation, keeping the no-monopole condition ($\nabla \cdot \mathbf{B} = 0$) in mind⁴⁸. The result for a steady density ρ_t in the wavefunction domain is:

$$-\nabla^2 \mathbf{B} = \mu_0 \left(\kappa g_1 \nabla \times \mathbf{j} - \frac{\kappa}{2} g_1^2 \nabla \times (\mathbf{A} \rho_t) \right) \quad (180)$$

At room temperatures, thermodynamics and high availability of energy create so many inelastic collisions that electrons will never have a coherent phase and the ρ_t term cancels out, leaving only the transport current. In a superconductor, the **complete opposite happens**.

In solid bodies, solitons exchange momentum and energy via field-mediated interactions⁴⁹. At some critical temperature T_c , this exchange hits some resonance condition (much like a high-Q-factor oscillator) and the low-energy electrons begin to lock onto each other’s phases, since they will have similar frequencies owing to insufficient energy. A full explanation lies outside the remit of the present work, but phenomenology has analogies in many coupled oscillator systems and macroscopic solitons as well[48]. There is no gradient in the distribution of phase among the solitons, which begin to act like one, or a “Bose-Einstein Condensate”. It is quite fortunate that the U(1) representation derivation does cover this case nicely, even if it does not fully capture the reason for the phenomenon (which ultimately involves spin, thus SU(2), left for future work).

The wavefunctions all become coherent, rendering ρ_t a macroscopic and observable phenomenon. If n_s carriers of charges’ worldtubes intersect on average over some position, then this will be the ρ_t value at that point. In the steady state of this resonance condition, due to dispersion and diffusion, there will be no macroscopic soliton density gradient or twist across the superconductor—all carriers would have phase locked. As such, ρ_t will

⁴⁷A detour could always be taken, fixing the Klein–Gordon equation parameter and obtaining it from the room-temperature constraint, but there is elegance in writing the derivation out this way.

⁴⁸This condition is derived from the Bianchi identity, which remains true even with these modifications.

⁴⁹Normally one distinguishes “photons” and “phonons” but both are abstractions of the field or the lattice.

be constant and can be pulled out from the cross product in the last term of equation (180). Applying (42) and the same principle:

$$\mathbf{j} = \rho \nabla \theta \Rightarrow \nabla \times \mathbf{j} = \nabla \times (\rho_t \nabla \theta) = \rho_t \nabla \times (\nabla \theta) = 0 \quad (181)$$

This leaves the decay equation:

$$\nabla^2 \mathbf{B} = \frac{1}{2} \mu_0 \kappa g_1^2 \nabla \times (\mathbf{A} \rho_t) \quad (182)$$

Since $\mathbf{B} = \nabla \times \mathbf{A}$:

$$\nabla^2 \mathbf{B} = \frac{1}{2} \mu_0 \kappa g_1^2 \rho_t \mathbf{B} \quad (183)$$

This is a screened Poisson equation. With physically appropriate boundary conditions, the magnetic field vanishes in the bulk. The screening length is:

$$\frac{1}{\lambda_L^2} = \frac{1}{2} \mu_0 \kappa g_1^2 \rho_t \quad (184)$$

The solution then, at some boundary at $z = 0$ is:

$$B_z = B_0 e^{-x/\lambda_L} \quad (185)$$

This exponential decay gradient within the superconducting material relative to the magnetic field that exists outside of it is the measurable **Meissner effect**. This was discovered by Walther Meissner and Robert Ochsenfeld in 1933. Hence, λ_L is experimentally observable. The remaining terms are constant, which tells us:

$$\frac{1}{\lambda_L^2 \rho_t} = \frac{1}{2} \mu_0 \kappa g_1^2 = \text{constant} \quad (186)$$

Here is a consistency check: since the entire superconductor is in a phase-lock condition, the only free density left is the charge carrier density, so again $\rho_t = n_s$ and it must be constant (at least within that model) since the penetration depth is likewise constant across all the boundary of the conductor. The penetration depth was measured experimentally and determined to match the London equations [49, 50, 51, 52]. This value could be used directly to determine the coupling strength κ , or the equivalent penetration depth model[53] could be used directly⁵⁰:

$$\lambda_L^2 = \frac{m}{\mu_0 n_s q^2} \quad (187)$$

Substituting (184) and the above:

$$\frac{\mu_0 n_s q^2}{m} = \frac{1}{2} \mu_0 \kappa g_1^2 \rho_t \Rightarrow \kappa g_1^2 = \frac{2q^2}{m} \frac{n_s}{\rho_t} \Rightarrow \boxed{\kappa g_1^2 = \frac{2q^2}{m}} \quad (188)$$

Now the coupled system is completely specified. Applying (175) and (188) yields the following coupling constants:

$$\boxed{g_1 = \frac{2q}{\chi}}, \quad \boxed{\kappa = \frac{\chi^2}{2m}} \quad (189)$$

⁵⁰In the original London model, a London parameter Λ was used rather than n_s .

Returning now to the Maxwell equation (168) and using the above relations, a covariant form valid when considering superconductivity can be derived:

$$\frac{1}{\mu_0} \partial_\mu F^{\mu\nu} = J^\nu + \frac{q^2}{m} \rho A^\nu \quad (190)$$

Rewriting (179) with these constants:

$$\nabla \times \mathbf{B} = \mu_0 \left(\frac{q\chi}{m} \mathbf{j} - \frac{q^2}{m} \mathbf{A} \rho \right) = \mu_0 \mathbf{J}_s \quad (191)$$

Here a new variable $\mathbf{J}_s$ has been defined as the supercurrent. A nicer result awaits if (42) is used and the density identified with n_s for both terms:

$$\mathbf{J}_s = \frac{n_s q}{m} (\chi \nabla \theta - q \mathbf{A}) \quad (192)$$

In the ground state of the superconductor, $\nabla \theta = 0$, therefore:

$$\boxed{\mathbf{J}_s = -\frac{n_s q^2}{m} \mathbf{A}} \quad (193)$$

The “un-derivable” London equation was derived without any postulation. □

11.2.3 Alternative Derivation

The coupling term g_1 can be measured from the hydrogen spectrum after χ is measured in a number of ways. If this were done, κ would be defined by the room-temperature condition, bypassing the entire London equation derivation. Both routes lead to the same conclusion, even if the alternate route is shorter though less insightful. The key takeaway is how consistent the coupled macroscopic-microscopic equations are.

11.3 Flux Closure



[To be completed in the final revision of this paper. Here I will demonstrate how the spinless soliton creates the same “magnetic flux quantum” in superconductors due to

phase coherence, but from the phase function only and because of its undefined nature when $\rho = 0$, much like a vortex integral, $\oint \nabla\theta \cdot d\mathbf{l} = 2\pi n$. It was taken out from the paper to meet the deadline and will be added next year.]

11.4 Fraunhofer Lines and the Hydrogen Spectrum

The hydrogen atom is the ultimate test for any ontological theory. As of 2025, none of them survive consistency. For example, the cutting-edge ontological framework of Bohmian mechanics interprets the wavefunction as a pilotwave on whose phase gradients a point-particle rides. The framework fails on the fundamental orbital in the s sub-shell, where there is no phase gradient and Bohmian mechanics predicts the particle to be stationary. Where is the particle stationary? The location is not defined. Yet, the electron must pass through the nucleus due to the wavefunction density and the Lamb corrections required to obtain a precise energy. These corrections are results of the electron-nucleus interaction.

The point particle was rejected early in the paper as not only logically inconsistent but yielding mathematical singularities that cannot be removed. With the options now narrowed to solitons alone, the task becomes demonstrating what the ontological picture says about electron orbits. Understanding the entire context of the problem at hand is imperative, and since the Schrödinger equation has already been derived, no further mathematical derivations remain to overcome. Only qualitative explanations and contrasts remain, with one exception: transition probabilities, discussed below but treated fully elsewhere.

11.4.1 Unexplained Spectral Bands

A mysterious phenomenon was discovered by William Hyde Wollaston in 1802 and then later by Joseph von Fraunhofer in 1814: the light spectrum of the sun was actually missing certain wavelengths! Today, these wavelengths are called Fraunhofer lines, observable in figure 5. These were collected through experimental measurements, their discovery predating much of the machinery that could explain them. Once armed with Maxwell's equations, investigators went off to work and yet none could account for their origin.

Maxwell's classical field theory that assumed a homogeneous macroscopic medium failed to explain the discrete pattern of spectral lines. The Fraunhofer lines did not merely indicate dispersion of light but a significant level of absorption in narrow non-continuous bands. Whatever was causing this absorption could not be a continuous substance. Back then, people only considered continuous charge distributions—the electron had not yet been discovered by J. J. Thomson. Prior to the development of Maxwell's equations, Gustav Kirchhoff and Robert Bunsen discovered in 1859 that heating gases produced observable emission lines. The same lines that were missing on the sun corresponded to lines emitted by gases. Spectrochemical analysis was born, accelerating developments in chemistry.

Speaking of schools, a Swiss schoolteacher gave the most significant clue into this mystery, much later in 1885. While not occupied with teaching, Johann Balmer spent his off hours doing numerology on the hydrogen spectral lines. Balmer was convinced the pattern was not just useful for identifying materials but had a structure to it that could serve as a clue. Three years later, the Swedish physicist Johannes Rydberg generalized Balmer's work, studying hydrogen, singly ionized helium, and doubly ionized lithium—all

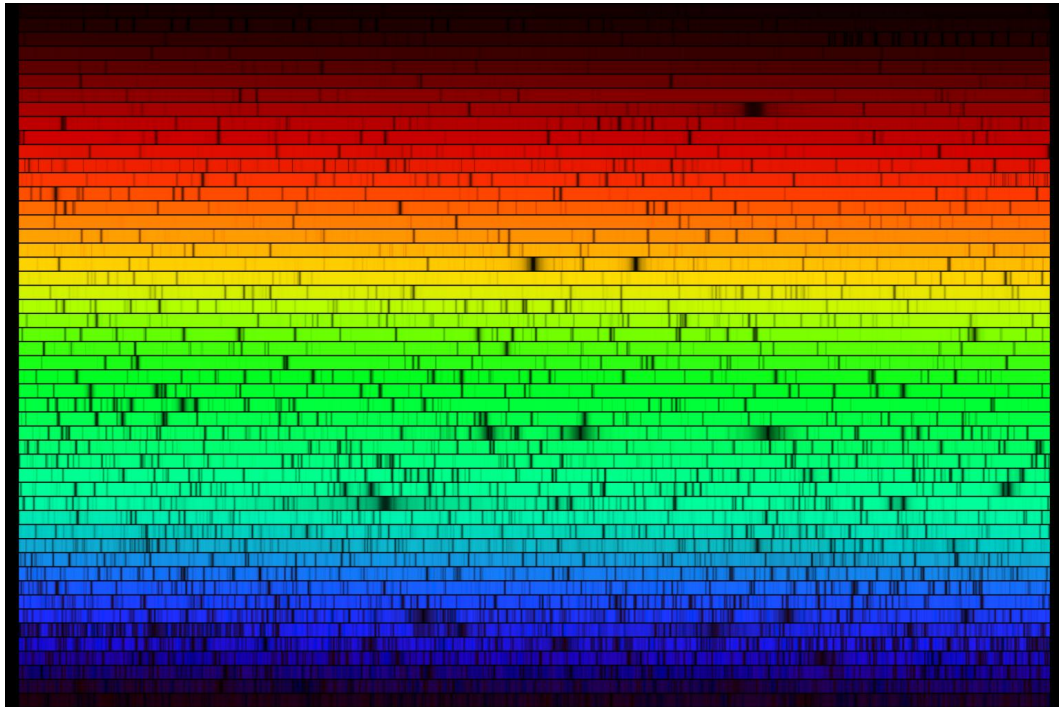


Figure 5: Solar spectrum showing Fraunhofer absorption lines.

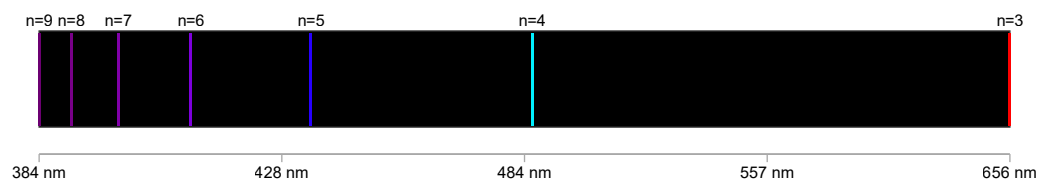


Figure 6: Hydrogen and other single-electron elements had a pattern to their emission lines. These are the $n=2$ Balmer series.

of these have only one orbiting electron. Through studies of spectra like the one in figure 6, Rydberg realized that there was indeed an underlying pattern.

Ionized helium just happens to share spectral lines with hydrogen and, from this and other observations, Rydberg discovered specific underlying **integers** (n) with transitions between them. The Rydberg formula for the line number indexed by integers n and m (with $n < m$) for hydrogen was:

$$k_{nm} = \frac{1}{\lambda_{nm}} = R_H \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \quad (194)$$

The Rydberg formula would prove to be revolutionary, yet only decades after Rydberg discovered it experimentally. The formula and number Rydberg discovered was actually universal, if adjusted for reduced mass and atomic number of single-electron ionized species⁵¹.

11.4.2 Bohr–Sommerfeld Model and Heisenberg’s Matrix Mechanics

Arthur Erich Haas first explained the Rydberg formula in 1910, but his unfortunate choice of J. J. Thomson’s obsolete plum pudding model led to a discrepancy by a multiplicative factor of 8. His idea was ridiculed by his peers⁵², but one scientist, Niels Bohr, paid attention and took Haas seriously. Bohr simplified Haas’s model to exclude the surface waves it posited.

Niels Bohr’s idea was beautifully simple when de Broglie’s interpretation is considered: Take a string held from both ends and think about its fundamental mode. The string has to meet the boundary conditions, namely, it cannot be in motion at the ends and needs to be held in position. Suppose one sticks the string together, forcing it to meet its own boundary condition. This mirrors the S^1 construction for the electron soliton’s phase—after one full oscillation, the phase must return to its starting value. This provides a closure condition and produces a discrete fundamental mode and overtones above it. Soon, the periodic table would reveal spherical harmonics, and to account for this, Sommerfeld proposed a now obsolete model that conserved ellipticity, allowing for elliptical orbits with the same closure condition.

One thing that still makes no sense in such models is why the electron should not radiate as it orbits its nucleus. The electron would need to accelerate towards the nucleus at all points of its orbit. Were the electron a point particle, continuous radiation would necessarily follow (as predicted by Larmor’s formula) and energy loss would drive an inevitable spiral into the nucleus. That question remains unanswered today; rather, it is wrapped around a formalism: Only eigenstates are measured in the bound state of an electron, so the electron does not radiate. A circular argument mistaking the epistemic with the ontological. This holds true in QED as well, which focuses on high-energy scattering and perturbation corrections, inheriting everything from Quantum Mechanics.

The Bohr model was severely limited: It predicted the correct format for Rydberg’s formula, but despite many attempts⁵³ could not consistently explain the intensities of emissions and absorption. That required Heisenberg’s matrix mechanics paper [54], which offered a very abstract solution involving Fourier transforms and requiring transitions

⁵¹Without adjusting for the Lamb shift and other relativistic effects such as the Darwin term.

⁵²Ernst Lecher called the idea a carnival joke.

⁵³Notably by H. A. Kramers who could obtain only relative intensities from Bohr’s correspondence principle, though the intensities were only correct for large orbits.

through an intermediate state to end up with the same energy. The paper is considered indecipherable by many physicists today; Heisenberg’s arguments centered on rejecting what cannot be measured experimentally and was ground zero for the death of ontology in physics. Nevertheless, the method produced the correct spectra and Wolfgang Pauli successfully analyzed the entire hydrogen atom with matrix mechanics by exploiting its hidden $SO(4)$ “symmetry” (conservation of ellipticity in disguise). Pauli concluded the process was cumbersome, and declined to use it to derive the intensities of absorption and emission.

A superior formulation was needed, and that is where de Broglie and Schrödinger stepped in with wave mechanics.

11.4.3 de Broglie, Schrödinger, and Their Wave Equation

“The damned Göttingen physicists now employ my beautiful wave mechanics for calculating their shitty matrix elements” - Schrödinger

de Broglie had an idea, almost identical to what is discussed here: the electron was a matter wave. De Broglie recognized that the electron must have an intrinsic frequency that shifts up, not down, when Lorentz boosted. De Broglie based his entire physics thesis on this idea [55] and shared it with Peter Debye and Erwin Schrödinger who were together in a joint seminar at ETH in Zurich, Switzerland back in 1925—a century ago. Back then, Schrödinger was a professor at the University of Zurich and liked de Broglie’s idea. Noting this, Debye asked Schrödinger to present de Broglie’s idea: *“Schrödinger, since you’re not working on very important problems right now, why don’t you tell us sometime about that thesis of de Broglie, which seems to have attracted some attention”*.

After presenting de Broglie’s idea, Debye was not exactly impressed and casually remarked that he thought the idea was childish. To deal properly with waves, one needed a **wave equation**. Schrödinger went off for just a few weeks, and returned with his wave equation, triumphantly declaring:

“Just a few weeks later he gave another talk at the workshop which began: ‘My colleague Debye suggested that one should have a wave equation; well, I have found one!’”

Over a series of extremely detailed, beautifully written papers [56, 57, 58, 59] Schrödinger formulated his wave mechanics theory. He then demonstrated that his theory is equivalent to Heisenberg’s matrix mechanics [60]. Having read all of Schrödinger’s papers, the depiction that he simply “guessed” the wave equation is inaccurate. Schrödinger cornered the problem, even if he did not prove it because he did not have a complete ontological picture of what he was working with.

Nevertheless, Schrödinger’s work gained substantial attention that year, and the ingenious English physicist Paul Dirac later used Schrödinger’s wavefunctions to derive the elusive intensities, which were equivalent to transition probabilities [61]. Today that equation is called “Fermi’s golden rule,” which is rather unfair.

A problem loomed over all of this: interpreting what ψ actually was. Here the 1927 Solvay conference in Brussels comes into focus. Two camps debated: the ontological camp, led by Schrödinger, de Broglie, and Einstein, against everyone else who sided with Heisenberg and his measurement-focused ideology. History testifies to the outcome: the majority won, and ontology died. The wavefunction was relegated to a probability distribution via Born’s rule, which was demonstrated to be trivially true by construction (the wavefunction is literally a joint manifold combining representational density and

phase). To that end, soliton mechanics⁵⁴ will be applied to this problem.

11.4.4 Solving the Hydrogen Atom

The Schrödinger equation has already been derived. Observations from any experiment so far could set χ , g_1 , or both—the hydrogen spectrum itself yields a combination of these values. For now, the values remain undetermined. The following form of the Schrödinger equation governs the representation of a soliton in an electrostatic field ϕ :

$$-\frac{\chi^2}{2m}\nabla^2\Psi + \frac{1}{2}g_1\chi\phi\Psi = i\chi\frac{\partial\Psi}{\partial t} \quad (195)$$

A linear equation with a Helmholtz spatial part. Inside an energy well, the equation produces discrete solutions with characteristic numbers n . These are called eigenstates and are resonance conditions, whose meaning will be explained in the next subsection. For now, the Fourier transform of the previous equation in time is taken, producing $\bar{\Psi}$. Applying the time-only Fourier transform replaces the partial time derivative with a frequency product: $\partial_t \rightarrow -i\omega$. This is not a “Hilbert operator,” but a basic property of Fourier transformations. Therefore:

$$-\frac{\chi^2}{2m}\nabla^2\bar{\Psi} + \frac{1}{2}g_1\chi\phi\bar{\Psi} = \chi\omega\bar{\Psi} \quad (196)$$

Applying the soliton frequency relation $E = \chi\omega$ yields the time-independent Schrödinger equation, whose arrival is now celebrated:

$$\boxed{-\frac{\chi^2}{2m}\nabla^2\bar{\Psi} + \frac{1}{2}g_1\chi\phi\bar{\Psi} = E\bar{\Psi}} \quad (197)$$

This produces spherical harmonic solutions when the energy is below zero relative to the system’s “vacuum potential” (potential at infinity). Essentially, the soliton is trapped in a well. By solving this problem, a set of eigenstates with eigenvalues equal to energies is obtained (essentially, what appears on the RHS as an eigenvalue problem). Subtracting these energies reproduces the transition energies from the Rydberg formula. A textbook calculation then yields a link between the Rydberg constant and the coupling values:

$$R_H = \frac{me^2g_1^2}{256\pi^3\epsilon^2c\chi} \quad (198)$$

Then, by fixing $\chi = \hbar$ through other experiments, one can obtain g_1 , or by fixing $g_1 = 2e/\hbar$ one can obtain χ . If both have been fixed, then this serves as a verification of the model.

But obtaining eigenvalues and eigenstates does not really provide a physical understanding of what is going on; the approach is essentially a Hilbert space formulation. The biggest problem of all in even that Hilbert space formulation is that the electron has a non-zero probability of being right in the middle of the nucleus in the ground state. Most textbooks gloss over this fact, yet it demonstrably defeats the point-particle and rigid-body ontological picture of the electron. Thankfully, soliton mechanics offers a valid, consistent, and understandable explanation of what is happening here and why solving this equation works.

⁵⁴It is noteworthy that soliton mechanics is almost identical in spirit to de Broglie’s double solution attempt, yet manages to complete it.

11.4.5 Soliton Mechanics in the Hydrogen Atom Context

Electrons can be categorized as bound and unbound, with numerous orders of bindings. An electron can be ‘free’ inside of a conductor, for example, yet still bound to that conductor. At the extreme end, it can be completely free in the void of space, in which case its rest frame makes complete and coherent sense. Yet, as established previously, the electron is a soliton like every particle. Lorentz invariance must hold since the particle is ultimately a field excitation much like a tornado, rather than a distinct ontological category such as a point-particle singularity. Yet it has an extended body so it flows through spacetime in a worldtube. The sole route to solve such a problem is through fluid dynamics, but that requires the restorative force that keeps the soliton together. This equation is not available, but the dispersive property of the soliton is known and a wavefunction that governs this dispersion has been derived.

Can one make do without the restorative form of the equation of motion? When considering bound electrons, one most certainly can. Only one observation is needed: The electron does not radiate when in a bound state, only while transitioning between states. This makes the puzzle especially acute for the point particle, because a point particle *must* accelerate to maintain an orbit. Yet, in the soliton model, the assumption proves invalid. In the ground state of a soliton, the soliton has a coherent phase within itself and the system it is in—after all, it is a field excitation—much like everything else around it (including the proton). Any deviation from the steady state is energy ready to be radiated away.

When analyzing the electron’s rest frame, the assertion was made that interference in the near field causes radiation to be emitted in the far-field. This does not extend to complex systems with multiple bodies as there may be steady-state absorbers and emitters. Instead, what could be asserted is that any perturbation from the steady state will require a restorative force to do work on the soliton, and the result of that is energy loss. The steady state is therefore defined when the restorative force balances out the dispersion, an important realization.

Within this state, the electron carries a constant energy and exists as a world tube that encompasses an **elastic deformation of its rest frame across spacetime**. When the electron transitions between these states, it must either absorb or emit the exact energy needed to step through these states, otherwise the states will not be stable and the electron would radiate whatever energy it receives. This creates the energy selection rule observed as energy eigenstates. The reader should remember that the spinless electron soliton is represented by a phase gradient along with an energy/mass density. Internally, the energy circulates at light speed c ; in a sense, velocity and acceleration are abstractions on top of this complex object. In that sense, the restorative force moves the electron through boost frames in spacetime, essentially deforming it in the proton’s worldtube rest frame.

This may be difficult to comprehend immediately, so two contrasting examples will make the picture concrete.

Consider the $n = 2, l = 1, m = 0$ state of hydrogen. This eigenstate carries an “angular momentum”; the electron is found strictly outside the nucleus, at the most probable distance of $4a_0$, where $a_0 = 0.529 \text{ \AA}$ is the Bohr radius. The point-particle framework easily pictures this as a planetary orbit—but the picture quickly disappears when considering the ground state. Suppose this same electron is disturbed from its excited state and returns to the ground state, emitting the excess energy, i.e., returning to $n = 1, l = 0, m = 0$. Where is the electron now, on average? First observe that

the electron has no angular momentum in this state and so cannot orbit the nucleus; it must go through the nucleus instead. As its motion is symmetric, that means the expected position of the electron is **the nucleus itself**. The orbit picture no longer makes sense and the only object that can pass through another in a medium without being permanently deformed is a soliton.

Nevertheless, the electron is most likely found at a distance of a_0 outside of the nucleus. It stretches out in both directions, and appears inside the nucleus too. If the electron were in motion, it would match the behaviour of a dipole antenna. Hence, it cannot be in motion. It is in a steady state of its world tube, where the phase gradient is determined by the wavefunction coupled to the proton's electric field. As it does not radiate, it does not move either—the soliton is simply stretched out across the distance of $2a_0$ from end to end and interacts with the proton producing the Lamb shift. This Lamb shift sets the scale where the restorative force becomes a significant contributor to the energy eigenvalue. The force clearly cancels out elsewhere and the dispersion alone suffices in defining the steady-state energy. Prior to QED, there was no quantum mechanics explanation for the discrepancy in energy. This picture is actually equivalent to that of QED but with a logically and mathematically consistent ontological perspective.

The same holds true for the “orbits”, only that the electron is stretched out in a spherical harmonic, or possibly a combination of them as long as they have the same energy. The orbits do not pass through the nucleus so do not experience a Lamb shift. It may be difficult to imagine, but despite such a strong deformation, the rest frame still looks exactly the same (only one soliton configuration exists as breathers are excluded). The rest frame's coherent phase distribution is simply deformed through space and time. The representation of this is a wavefunction. Laminar flow provides a useful motivating example. Under an ultra-laminar (Stokes) flow inside of a high-viscosity medium such as corn syrup, dyes may be added to visualize the flow. Rotate the corn syrup gradually enough and the dyed areas may appear to be completely mixed. Undo the rotation just as gradually and the original configuration will return to normal. To aid in visualizing this, screenshots from a popular demonstration are included in figure 7.⁵⁵



Figure 7: Demonstration of laminar flow reversibility in a Taylor–Couette apparatus. **Left:** Colored dye droplets are injected into a viscous fluid (corn syrup) confined between two concentric cylinders. **Center:** After several rotations of the inner cylinder, the dye appears thoroughly mixed. **Right:** Reversing the rotation by the same number of turns “unmixes” the fluid, returning the dye to approximately its original configuration. This reversibility is possible because the low-Reynolds-number flow is dominated by viscous forces, and molecular diffusion occurs on a much slower timescale than the demonstration.

In the Stokes flow, the origin rest frame state is never modified (though it does dif-

⁵⁵https://youtu.be/p08_KlTKP50

fuse), space is *twisted* instead. The soliton case works identically, but the entire soliton participates and finds an equilibrium state which will be the eigenstates of its dispersion equation, the Schrödinger equation in the low-energy limit.

11.4.6 Line Broadening

The steady state picture does not explain the energy transitions, the spectral frequency, nor absorption and emission process intensities. In Quantum Mechanics, an elaborate picture is painted of electrons “emitting” and “absorbing” photons and QED takes this to the extreme with the field itself being reduced to these forms of “interaction”. In a continuous field with soliton excitation, interaction is continuous, so how can transitions be discrete and why do they happen in discrete frequencies? The answer is actually in one of the less discussed aspects of electron energy transitions, namely line broadening.

An electron transitioning between two bound states will emit a wave packet with the following frequency and energy:

$$\Delta E = E_f - E_i = \hbar\omega_f - \hbar\omega_i = \hbar\delta\omega \quad (199)$$

Does the electron emit a wave packet with exactly this frequency? It does not; there is “natural line broadening” away from this mean energy. The usual (and incorrect) pedagogical position is to attribute this to the Heisenberg uncertainty principle. Aharonov and Bohm showed the claim is false through their counter-example in 1961[62, 63]. The line broadening does not happen due to Landau and Peierls’ mathematical formalism, but because of the finite lifetime of the excited state and atom-field coupling. Incidentally, quantum electrodynamics takes the same position.

To make this concrete, in continuous waves, radar pulses, or any finite wave train, the spectrum of $w(t) = e^{-t/2\tau}e^{i\omega_0 t}$ will have spectral width $\Gamma \propto 1/\tau$. As a continuous process of deceleration or acceleration of the bound electron, the faster this happens, the greater the spread in the spectrum. The line broadening needs no further explanation for solitons as they are a form of continuous wave. The faster it happens, the broader the spectrum will be.

11.4.7 Emission Frequencies

That explains line broadening, but can one possibly explain why the electron should emit one particular frequency? Quantum Mechanics takes this for granted through the photon postulate, and actually even those who do not pay regard to the photon models take it for granted without an ontological explanation. The soliton ontology offers a reason, and it has everything to do with the electron’s rest frequency and how it changes under a boost.

The start and end frequency in (199) is physical in the soliton picture, but then so is the continuous wave process described in the earlier section. Not all of the soliton will accelerate together, as it will be spread out in spacetime according to its phase and density relationship. As such, there will be nonlinear mixing between the two eigenstate frequencies too. In the end, the electron gains the difference or the field does, but they do so through a continuous mixing process that generates a wave packet of a certain frequency. The various types of emissions are then handled by selection rules which are an artifact of the nonlinear restorative force of the electron imposing its dispersion relations.

This may be difficult to see due to the inherent self-interaction of the field, but will be much clearer with the Compton scattering experiment later in the paper because of the peculiar setup of that experiment. The complete treatment is deferred until then.

11.4.8 Transition Probabilities

An analysis of transition probabilities lies beyond the present scope, as the topic deserves treatment of its own. Instead, the following reviews Hilbert-formalism-derived methods for constructing these probabilities, and examines which assumptions come baked into them. Within Hilbert formalism, inner products between two eigenstates define only occupation probabilities among superposition states. Obtaining probabilities for transitions between states—the physics that actually produces spectral lines—requires turning to perturbation theory and invoking several assumptions lying outside pure Hilbert space structure. In Dirac’s 1927 formulation, which was actually the foundation of Quantum Electrodynamics [61], the transition probability from state i to state f is calculated by applying the following equation:

$$\Gamma_{i \rightarrow f} = \frac{2\pi}{\hbar} \left| \langle f | \hat{H}' | i \rangle \right|^2 \rho(E_f) \quad (200)$$

The key component here is the transition amplitude: the overlap between the target state $\langle f |$ and the *time-perturbed* initial state $|i\rangle$. Note that the bare Hilbert overlap of stationary eigenstates vanishes,

$$\langle f | i \rangle = 0 \quad (f \neq i) \quad (201)$$

so the relevant quantity is effectively the perturbation-weighted overlap. The approximation H' is only valid in the weak-coupling limit.

Yet, the most significant component of this equation is $\rho(E_f)$, which represents the continuous spectrum of electromagnetic field modes that can absorb the electron’s emission. To make the Hilbert machinery work, Dirac approximated this continuum with discrete modes in a box. He stated that this step proves crucial because both the initial and final bound electron states are discrete, yet the electromagnetic field into which energy flows has a continuous mode spectrum. In free space, his quantization of the field produced a density with normalized volume V , taking the form:

$$\rho(\omega) = \frac{\omega^2 V}{\pi^2 c^3} \quad (202)$$

Without the discretization of the continuum, “Fermi’s golden rule” would not yield a transition rate at all—rather, the equation would produce indefinite Rabi oscillations between states [64]. The first problem with Dirac’s equation is that the volume V is arbitrarily chosen and obviously cannot represent the actual medium it models. This perturbation method is a controlled approximation with known validity conditions: weak coupling, long times, and a genuine continuum of final states. While it yields correct answers within these limits, this instrumental success does not mean it represents the correct physical picture of what actually occurs during a transition.

There is another subtly presented but massive ontological problem here that most miss. Dirac uses a theory-laden energy density for the medium by assuming a linear relationship between frequency and energy. So far, this has only been demonstrated for the electron soliton due to perfect frequency lock in the rest frame. Here lies the

most important point of the present work: the universality of the relation $E = \hbar\omega$ was not assumed, and for a good reason! The field energy is calculated by an energy density integral $\int \left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \right) dV$ in electromagnetism, which has no frequency dependence. Yet, because all interaction with the field is measured with electrons, is this distinction directly observable to us?

The justification actually lies in another experiment: Compton scattering. Many seized upon Compton's results to force the "photon" (or "quanta") ontology, resurrecting the defeated corpuscular theory in the process. Yet that threw out the baby with the bathwater—the idea being that Compton scattering "proved" photons were not merely computational aids but ontological entities, and that light was discrete in nature. That was not demonstrated, and a careful analysis of the experiment would reveal that the discreteness came from the electron's responses and the source's wave packet formation. Nevertheless, this theory-laden assumption is now everywhere and has developed into the 'second quantization' of the entire field. The assumption additionally creates infinities in the computation—both ultraviolet and infrared—much like the singularity of the Coulomb field for a point charge: symptoms of an ontology pushed beyond its domain of validity.

In light of this, the Compton scattering subsection approaches this differently: rather than computing transition probabilities, the continuous form of interaction between field and soliton (extended in nature) is derived. Illuminating that foundational problem may offer a step towards a framework free of the computational methods Dirac prescribed, methods that eventually diverge in the UV and demand 'renormalization' techniques Dirac himself detested as illegitimate.⁵⁶ Deep research into the subject has revealed that this is the only known formal derivation of Compton scattering from purely classical field-soliton interaction. As such, this discussion is postponed until Compton scattering is covered.

11.5 Photoelectric Effect

With the hydrogen spectrum and Fraunhofer lines covered, the photoelectric effect becomes trivial to explain. First observed by Heinrich Hertz, when a metallic material is subjected to incident light, it will only eject electrons after a certain threshold of frequency is passed. After that threshold is passed, the ejection rate scales with light intensity. This phenomenon was measured and confirmed by Philipp Eduard Anton von Lenard (Hertz's assistant), and he accurately measured the intensity relationship. In 1905, Einstein showed that the kinetic energy of the ejected electron⁵⁷ varies linearly with the frequency in excess of the threshold, and that the constant of linearity was Planck's constant, h . Unfortunately, Einstein exceeded what was required to explain the phenomenon and proposed that light is broken up into discrete "quanta" (later renamed "photons"); this idea would soon infect physics even though even the experimenter who verified Einstein's prediction (Robert Millikan) opposed his interpretation.

⁵⁶To quote Dirac himself, "Most physicists are very satisfied with the situation. They say: 'Quantum electrodynamics is a good theory and we do not have to worry about it any more.' I must say that I am very dissatisfied with the situation because this so-called 'good theory' does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it is small—not neglecting it just because it is infinitely great and you do not want it!"

⁵⁷Amusingly, the photoelectric effect was discovered 10 years before electrons were.

In the soliton mechanics picture, the photoelectric effect is a trivial extension of the bound state transition into the ‘vacuum’ field. Light absorption by the electron is a non-linear process where continuously incident light converts completely to a boost. As the electron’s frequency depends on boost frame energy, this directly explains the proportionality. Incident light below the emission threshold will drive an off-resonance oscillation that decays in the material and does not accumulate into an ejection of an electron, similar to the Thomson scattering case but for bound rather than free electrons.

The work function characterizing each material is simply the binding energy of the highest occupied electron state—the soliton equivalent of the hydrogen ionization energy, but for the collective potential well of the metal lattice. The simultaneous intensity proportionality and frequency dependence above threshold demonstrates two modes of coupling between the field and the ensemble of electrons. The intensity can be thought of as the vector potential A^μ and the density of electrons in the material as ρ ; both the nonlinear mixing mode and the bilinear proportionality mode are observed.

With this experiment χ can be extracted as the coefficient of proportionality between the kinetic energy of the electron (measured as a voltage) and the incident light. A similar but superior argument was developed by Lamb and Scully [15] so there is not much further to add to this subsection.

11.6 Thomson Scattering

Thomson scattering was the first theoretical treatment of light scattering by free electrons, developed by the English physicist J. J. Thomson shortly after his discovery of the electron. The apparatus that would make that discovery possible was invented by another English physicist, William Crookes. The Crookes tube was a high-vacuum discharge tube that produced cathode rays, generating both electrons and x-rays simultaneously⁵⁸ (Röntgen rays as they were called), though this was not known when they were invented. Thomson would use electrostatic and magnetic deflection to control the discharge, and detected what he called “corpuscles”, later renamed the electron to match the terminology of George Johnstone Stoney who anticipated their discovery.

On the back of the discovery of both electrons and x-rays, Thomson sought to experimentally deduce the behavior of their interaction. He predicted re-emission of x-rays from free electrons due to the electrons radiating in the field as they accelerated in harmony with the incident light. This was a continuous process, and the steady-state emission also underwent Doppler shifts and broadening. Because of the low energies involved, there is no frequency shift other than the Doppler shift. Ironically, despite being lower in energy than Compton scattering, and deemed “classical”, the mathematics behind Thomson scattering was quite involved.

Within the context of the present analysis, Thomson scattering operates entirely within the linear regime of the bilinear approximation to the unified field Φ . Thomson’s scattering equation was primarily concerned with the intensity of x-rays as a function of scattering angle, expressed through the differential cross-section:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}r_e^2(1 + \cos^2 \theta) \quad (203)$$

⁵⁸It is ironic that though Crookes invented his gas discharge tube, J. J. Thomson discovered the electron using them and Röntgen discovered x-rays.

where σ is the cross-section, r_e is the classical electron radius, Ω is the solid angle, and θ is the scattering angle.

For low energies, this equation worked well, and no frequency shifts occurred—only Doppler broadening from thermal motion. As such, any frequency shift against the incident radiation would break this model. As people examined this scattering model, they indeed began to detect frequency shifts (called “radiation softening”) as they increased the frequency of radiation. The frequency dependency was unexpected, exactly like in the photoelectric effect. This would ultimately inspire one American scientist, Arthur Compton, to put the Thomson equation through a thorough test and probe the true nature of the electron.

11.7 Compton Scattering

Arthur Holly Compton was a renowned American experimental physicist with a truly inquisitive mind. He and many others noted that Thomson’s formula did not work for high-energy (“hard”) x-rays. Furthermore, they detected an anisotropy in the cross-section, with greater intensity in the forward direction ($\theta = 0$) than the backwards side ($\theta = 180^\circ$). The formula predicted the same cross-section as $\cos^2(0) = \cos^2(180^\circ)$. For Compton, this was evidence that the electron had a radius around the wavelength scale he was using. Under this basis, he developed his theory and experimental methodology, proposing [65, 12, 66] three shapes of an electron that mirror the process of elimination undertaken early in the paper:

- A rigid shell of charge incapable of rotation.
- A flexible shell of charge capable of rotation.
- A flexible thin ring of charge that rotates.

The rigid shell produced a symmetrical cross-section. The flexible shell produced an anisotropy but a poorer quantitative result. Of these three, Compton favored the thin ring charge model on quantitative grounds.⁵⁹ Though the anisotropy caught his immediate attention, something else led him in another direction: As he varied the scattering angle, he detected an absorbability shift caused by softening of the scattered x-ray. This pointed to a variation in *frequency* that was entirely unanticipated by Thomson’s energy-only model. Compton was not the first to notice; Sadler & Mesham (1912), Gray (1913), and Florance (1914) had all observed the phenomenon[67, 68, 69]. All of them used indirect means to detect this change, namely the absorption coefficient of x-rays in lead and aluminum. These were technically the very first Compton scattering experiments.

Compton, nevertheless, would take that experiment to the next level and measure the wavelength offset directly, simultaneously with the anisotropy. The experimental apparatus is carefully illustrated in fig. 8 and consists of a Molybdenum K-alpha x-ray Coolidge tube⁶⁰ source, a collimating slit, a carbon target, a calcite Bragg deflector which acted as a spectrograph, and finally, an ionization chamber to record the intensity of the scattered x-ray. The setup allowed Compton to vary two angles, one to set the scattering angle

⁵⁹Compton was also obviously influenced by Parson’s magneton, as he had mentioned Parson’s work in his 1919 paper.

⁶⁰a hot cathode variant of the Crookes tube

of the x-ray, and another to scan the Bragg deflector, thereby constituting a spectrometer. This gave Compton a controllable θ , a fixed source frequency ω_γ , and a measurable scattered frequency ω'_γ . All that was left was to find the characteristic transfer profile of the electron dependent on these variables. Now, Compton was working with wavelengths because they mapped adequately to the experimental setup (observations from Bragg deflectors are wavelength correlations), but working with frequencies will reveal something significant and illuminating about this experiment that most miss.

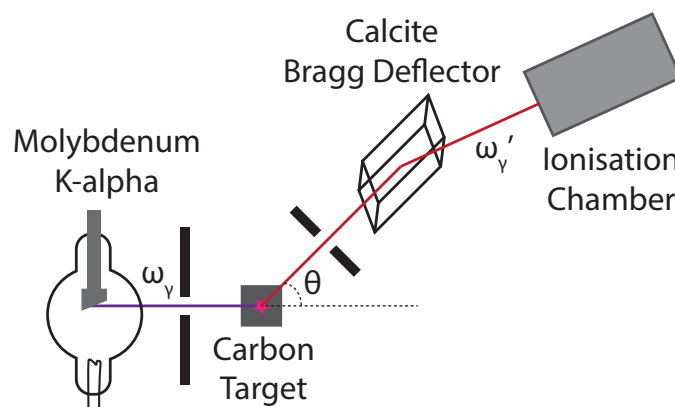


Figure 8: Schematic of Compton's experimental apparatus. A Molybdenum K-alpha x-ray source emits radiation at frequency ω_γ , which passes through a collimating slit before striking a carbon target. The scattered radiation at frequency ω'_γ is analyzed by a calcite Bragg deflector acting as a spectrometer, with intensity recorded by an ionization chamber. The scattering angle θ can be varied to measure the wavelength shift as a function of deflection.

The key finding was that the scattered wavelength (frequency) shift had an angular dependency and a profile that characterizes the electron in ways electromagnetism alone could not explain, at least during that time. Despite being a proponent of the ring electron, Compton reached out for Einstein's photon postulate and made the scattering phenomenon a kinematic inelastic process. By treating the light as though it was corpuscular, he obtained a result that matched his experimental setup. While many physicists saw this as the end of electromagnetism and what pedagogical treatments would later call "classical physics", some treated this conclusion as a reversion to the past debates during the Enlightenment. In particular, Bohr detested the corpuscular reversion because photons could not handle interference. For this reason, he created the rather brittle BKS theory, which comprised ensembles of electrons colluding to create the scattering phenomenon, an idea that was thoroughly defeated by the Bothe–Geiger coincidence experiment.

Schrödinger solved the puzzle in 1927, realizing that the electron, which had been scattered by the x-ray, had become exactly like another component of the experiment: a moving Bragg deflector. If indeed the electron's density in its worldtube formed a periodic function and it moved against the electromagnetic field, it would re-emit a wave of a different frequency. His paper was somewhat qualitative and was largely ignored: photons dethroned electromagnetism, leading to Dirac's formulation mentioned earlier in the transition probabilities section. The rest was history...

The problem with what exists today is that it compresses what has to be a continuous process (electrons cannot accelerate at an infinite rate) to an ‘event’, a continuous wave (gamma rays) to a ‘photon’, and of course an extended soliton to a point-particle. Is it any surprise that singularities appear in all of the calculations today? The kinematic formula derived by Compton works, but it conceals everything that happens during the electron-field interaction. A return to Schrödinger’s idea is necessary to reveal whether it has quantitative merit.

One experimental observation stands out: the electron is not annihilated by any of the gamma rays. Such behavior confirms what a soliton requires as a topological deformation of the medium—the only object capable of annihilating it is its own anti-soliton (the positron). The restorative force therefore keeps the electron whole in its rest frame throughout the collision. Now, Lorentz invariance does not hold in an arbitrarily accelerated frame, but the analysis here adopts the electron’s rest frame just prior to collision. Without a deeper model, stating exactly what happens to the soliton during collision remains impossible, yet nature permits a first-order approximation of what occurs right in the middle of it.

11.7.1 Definitions

Time to carefully define all the variables necessary for explaining the Compton effect: ω_γ is the incoming light frequency, ω'_γ is the scattered light frequency, θ is the scattering angle, ω is the scattered electron soliton frequency, and $\mathbf{k}$ is its wavenumber. That gives the scattered electron a K^μ vector of $(\frac{\omega}{c}, \mathbf{k})$. The goal is to find ω'_γ when the remaining variables are known. To simplify the mathematics, the starting interaction frame is taken to be the electron’s rest frame, in which case its 4-vector K_i^μ is $(\frac{\omega_0}{c}, \mathbf{0})$, i.e., $\mathbf{k}_0 = \mathbf{0}$.

Two wavefunctions are now defined, ψ_i and ψ_f , representing the starting and final state of the electron and applying the same method defined earlier:

$$\psi_i = \sqrt{\rho_i} e^{-i\omega_0 t} \quad (204)$$

$$\psi_f = \sqrt{\rho_f} e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)} \quad (205)$$

Neither ψ_i nor ψ_f can represent the soliton as it interacts with the electromagnetic field, and the nonlinear (and as will be seen shortly, necessarily nonlocal) Lagrangian and its form are not available. Nevertheless, a lowest order approximation of this collision/recoil interaction can still be derived by taking a nonlinear interpolation between these two wavefunctions, using complex mixing functions a_1 and a_2 that keep the transitional wavefunction ψ normalized:

$$\psi = a_1 \psi_i + a_2 \psi_f \quad (206)$$

11.7.2 Scattering Current

Equation (48) may now be used to calculate the 4-current (and thus both charge density and motion of the electron):

$$j^\mu = \frac{i}{2} \left[(a_1 \psi_i + a_2 \psi_f) \partial^\mu (a_1^* \psi_i^* + a_2^* \psi_f^*) - (a_1^* \psi_i^* + a_2^* \psi_f^*) \partial^\mu (a_1 \psi_i + a_2 \psi_f) \right] \quad (207)$$

Applying standard manipulations from complex analysis, the expanded product can be written compactly as:

$$j^\mu = |a_1|^2 j_i^\mu + |a_2|^2 j_f^\mu + j_{if}^\mu \quad (208)$$

where j_i^μ and j_f^μ are the equivalent 4-currents of the initial and final states, and j_{if}^μ is the cross-current:

$$j_{if}^\mu = -\text{Im} \left[a_1 a_2^* (\psi_i \partial^\mu \psi_f^* - \psi_f^* \partial^\mu \psi_i) \right] \quad (209)$$

For convenience, define $A = (\psi_i \partial^\mu \psi_f^* - \psi_f^* \partial^\mu \psi_i)$, and the phases $\phi_i = -\omega_0 t$ and $\phi_f = \mathbf{k} \cdot \mathbf{x} - \omega t$. Since $\partial^\mu \psi = \psi \left(\frac{\partial^\mu \rho}{2\rho} + i \partial^\mu \phi \right)$, A takes the form:

$$A = \psi_i \psi_f^* \left[\frac{\partial^\mu \rho_f}{2\rho_f} - \frac{\partial^\mu \rho_i}{2\rho_i} - i (\partial^\mu \phi_f + \partial^\mu \phi_i) \right] \quad (210)$$

Note the $\psi_i \psi_f^*$ factor in the integrand of Hilbert formalism and Fermi's golden rule. Additionally:

$$\psi_i \psi_f^* = \sqrt{\rho_i \rho_f} e^{i(\phi_i - \phi_f)} = \sqrt{\rho_i \rho_f} e^{i\Phi}, \quad \boxed{\Phi \equiv \phi_i - \phi_f = (\omega - \omega_0)t - \mathbf{k} \cdot \mathbf{x}} \quad (211)$$

To further express A in a meaningful and compact way, define the following four vectors:

$$R^\mu = \frac{1}{2} \partial^\mu \ln \left(\frac{\rho_f}{\rho_i} \right) \quad (212)$$

$$S^\mu = (\omega_0 + \omega, \mathbf{k}) \quad (213)$$

Now the scattering current can be reassembled:

$$A = \sqrt{\rho_i \rho_f} e^{i\Phi} [R^\mu + i S^\mu] \quad (214)$$

$$\zeta = \arg(a_1) - \arg(a_2) \quad (215)$$

$$j_{if}^\mu = -|a_1||a_2|\sqrt{\rho_i \rho_f} [R^\mu \sin(\Phi + \zeta) + S^\mu \cos(\Phi + \zeta)] \quad (216)$$

Exactly the form needed.

11.7.3 Mid-Scatter

Now consider the transition surface where the mixing between the pair of states is maximized, i.e., where $|a_1|^2 = |a_2|^2 = \frac{1}{2}$. At this halfway point, the soliton is equally distributed between rest and boosted configurations. The restorative force maintains continuity of the dynamic density $|\psi|^2$ throughout the transition, smoothly interpolating between ρ_i and ρ_f . At the intersection of these two distributions, $\rho_i \approx \rho_f \equiv \rho$ is expected: the initial state is isotropic while the final state undergoes Lorentz contraction along $\hat{\mathbf{k}}$, but at the spatial region where both configurations overlap with equal weight, their densities match.

This enables taking $R^\mu \approx 0$, eliminating the density gradient term. Moreover, $|a_1||a_2|\sqrt{\rho_i \rho_f}$ reduces to $\frac{1}{2}\rho$, which leaves the result:

$$j_{if}^\mu \propto -\rho [\cos((\omega - \omega_0)t - \mathbf{k} \cdot \mathbf{x} + \zeta)] \left(\frac{\omega_0 + \omega}{2}, \frac{\mathbf{k}}{2} \right) \quad (217)$$

Taking the timelike component of this 4-current reveals a density that is exactly as Schrödinger claimed: a continuous and moving grating, with a beat frequency of $\Omega \equiv \omega - \omega_0$, a k vector halfway to the final wavenumber (and thus momentum), and a frequency which is mid-way as well since:

$$\frac{\omega_0 + \omega}{2} = \omega_0 + \frac{\Omega}{2} \quad (218)$$

Furthermore, the result is a moving grating with a phase velocity $v_{ph} = \Omega/\mathbf{k}$, analogous to the photoelastic effect in acoustic-optics [70]. As the electron emits only one wave packet during this interaction, the average direction of motion will not change. Through continuity, it can be assumed that this moving grating will form smoothly over its motion, generating some envelope that starts without the grating pattern, develops the full grating pattern in the middle, and shows no grating pattern once scattered⁶¹.

11.7.4 Field Interaction

The mixing equation from initial to final wavefunction is now established; focus now turns to the incident light itself. X-rays and other high-energy waves usually originate from matter, arriving in packets. Nevertheless, for an instantaneous evaluation, only the section of the wave at maximum amplitude need be considered rather than the entire envelope. The incident plane wave's electric field:

$$\mathbf{E} = \mathbf{E}_0 \cos(\mathbf{k}_\gamma \cdot \mathbf{x} - \omega_\gamma t) \quad (219)$$

Poynting's theorem (derivable from Maxwell's equations), states that:

$$\frac{\partial u}{\partial t} + \nabla \cdot \mathbf{S} = -\mathbf{E} \cdot \mathbf{J} \quad (220)$$

where u is the energy density, $\mathbf{S}$ is the Poynting vector, and $\mathbf{J}$ is the current density. The result scales with what was calculated by a constant (see section 8 for details). This equation defines when electrons will radiate or absorb energy from the field. The complete dynamics are not available, but the dot product on the RHS can be worked out:

$$-\mathbf{E} \cdot \mathbf{J} \propto (\mathbf{E}_0 \cdot \mathbf{k}) \cos(\mathbf{k}_\gamma \cdot \mathbf{x} - \omega_\gamma t) \cos(\mathbf{k} \cdot \mathbf{x} - \Omega t - \zeta) \quad (221)$$

These are two cosines mixing against each other, with the beat frequency being Ω . The two frequencies can be broken up using trigonometry, $\omega_\gamma + \Omega$ and $\omega_\gamma - \Omega$. These will be two Fourier components that can either contribute to the energy density's change over time (i.e., the electron moving), or to outgoing radiation via the Poynting vector. As pointed out earlier, the electron does not disintegrate, so at no point can it provide any energy into the field if it is at rest. Equivalently, its rest frame frequency ω_0 is a conserved topological charge. As such, the outgoing wave that it emits will be restricted to the ω_γ branch, while the remaining energy will interact with the restorative force to place the electron in motion. In the end, the electron's frequency is up-shifted by Ω by definition

⁶¹Physicists are in a bind in terms of observability as the internal motion of the electron and its undulation of the x-ray is so rapid that equipment with zeptosecond resolution would be needed to truly probe what motion the electron takes through this scattering process. The envelope may reveal something about the restorative force or even the structure inside the electron if some of these deductions are incorrect. This kind of timescale remains in the realm of science fiction as electrons are used for all interaction and experimentation.

and part of the energy from the field is re-emitted as a down-shifted wave packet. In other words, the following relationship has been determined:

$$\boxed{\omega_\gamma - \omega'_\gamma = \Omega \equiv \omega - \omega_0} \quad (222)$$

Identify the wavenumber of the photon by expanding this branch only:

$$\cos(\mathbf{k}_\gamma \cdot \mathbf{x} - \omega_\gamma t) \cos(\mathbf{k} \cdot \mathbf{x} - \Omega t - \zeta) = \cos((\mathbf{k}_\gamma - \mathbf{k}) \cdot \mathbf{x} - (\omega_\gamma - \Omega)t) + \dots \quad (223)$$

Thus,

$$\boxed{\mathbf{k}'_\gamma = \mathbf{k}_\gamma - \mathbf{k}} \quad (224)$$

A pause here is warranted to appreciate that this result was arrived at without invoking kinematics, only Lorentz invariance and the soliton's conservation condition (i.e., charge conservation or its single rest frame frequency). Except for the energetic justification of branch selection, energy or momentum was not even mentioned, only frequencies and wavenumbers.

11.7.5 Dispersion Relations

The following dispersion relation for the electrons' aggregate four-wavenumber was previously derived from Lorentz invariance alone:

$$\boxed{\omega^2 = \omega_0^2 + c^2 |\mathbf{k}|^2} \quad (225)$$

In general, for any wave traveling in a medium with speed c , the dispersion relation is fixed with the linear form:

$$\boxed{\omega_\gamma = c|\mathbf{k}_\gamma|, \quad \omega'_\gamma = c|\mathbf{k}'_\gamma|} \quad (226)$$

With the last four boxed equations, the problem is now completely constrained and the desired function can be found.

11.7.6 Compton Scattering Equation

The remaining vector algebra follows quickly. First, handling the wavenumbers through (224):

$$\mathbf{k}'_\gamma = \mathbf{k}_\gamma - \mathbf{k} \Rightarrow |\mathbf{k}|^2 = |\mathbf{k}_\gamma|^2 + |\mathbf{k}'_\gamma|^2 - 2|\mathbf{k}_\gamma||\mathbf{k}'_\gamma| \cos \theta \quad (227)$$

Then, eliminating all the wavenumbers using each of the dispersion relations (225), (226):

$$\omega^2 = \omega_0^2 + c^2 \left(\frac{\omega_\gamma^2}{c^2} + \frac{\omega'^2_\gamma}{c^2} - 2 \frac{1}{c^2} \omega_\gamma \omega'_\gamma \cos \theta \right) \quad (228)$$

Now, eliminating the electron's final frequency ω , which is not directly observed by the experiment, via (223):

$$\omega = \omega_0 + \omega_\gamma - \omega'_\gamma \quad (229)$$

Which becomes the following when substituted into the previous relation:

$$(\omega_0 + \omega_\gamma - \omega'_\gamma)^2 = \omega_0^2 + (\omega_\gamma^2 + \omega'^2_\gamma - 2\omega_\gamma\omega'_\gamma \cos \theta) \quad (230)$$

Expanding and eliminating the common terms from the left- and right-hand sides:

$$\omega_0(\omega_\gamma - \omega'_\gamma) = \omega_\gamma\omega'_\gamma(1 - \cos \theta) \quad (231)$$

Rearranging:

$$\boxed{\frac{1}{\omega'_\gamma} - \frac{1}{\omega_\gamma} = \frac{1}{\omega_0} (1 - \cos \theta)} \quad (232)$$

The Compton scattering equation has been derived. Just not the one most are familiar with! Multiply the entire equation by $2\pi c$, and use $c = f\lambda$:

$$\lambda'_\gamma - \lambda_\gamma = \frac{2\pi c}{\omega_0} (1 - \cos \theta) \quad (233)$$

Recall that $E_0 = mc^2 = \chi\omega_0$ was derived. Equivalently:

$$\lambda'_\gamma - \lambda_\gamma = \frac{\chi}{mc} (1 - \cos \theta) \quad (234)$$

This completes the derivation. □

11.7.7 Measuring the Soliton Frequency ω_0

In the derivation, only frequency and wavenumber were utilized. Bragg deflectors deal with wavelengths but these are also directly convertible to frequencies. It was mentioned earlier that ω_0 could be measured, and here that promise is realized. First, the equation is rearranged like so:

$$\frac{\omega'_\gamma}{\omega_\gamma} = \frac{1}{1 + \frac{\omega_\gamma}{\omega_0}(1 - \cos \theta)} \quad (235)$$

This expression is a transfer function but in terms of frequency rather than intensity or amplitude, with dependency on incident frequency as well. In the low frequency sector, the Thomson scattering frequency independence is somewhat restored. It is only as the electron's frequency is approached that the scattering radiation shifts.

That proves the electron's frequency is **experimentally observable**, but the standard forms of the equation conceal this. An experiment utilizing polychromatic gamma rays via synchrotron radiation could sweep this transfer function to produce a logarithmic chart with a pole exactly at this frequency. This ontology has greater legitimacy than speaking of a Compton wavelength or radius without a proposed electron structure. Indeed, this corresponds to the de Broglie frequency and the key inspiration for his matter-wave hypothesis. Values from three experiments are now integrated to get a crude estimate of this value and compare it against the theoretical value $\omega_0 = \frac{mc^2}{\chi}$. Because the dataset is sparse, the Compton scattering equation will be transformed into one that determines ω_0 :

$$\omega_0 = \frac{1 - \cos \theta}{\frac{1}{\omega'_\gamma} - \frac{1}{\omega_\gamma}} \quad (236)$$

For convenience, Hertz is used rather than radial frequency. Two experiments with publicly available data, conducted with relatively good quality equipment, were chosen, one by a physics student and the other by a professional nuclear engineer. These experiments use two radioactive gamma ray sources near the predicted frequency of the electron.⁶² Obviously, higher precision experiments are possible, but the aim is to demonstrate the relatively accessible measurability of ω_0 . See table 1 for the computed values; energies in the source (usually quoted in MeV) have been converted into frequencies. The dataset was larger but some representative data that were not subject to systematic errors (through angle calibration) were picked for purely demonstrative purposes.

Table 1: Experimental Compton scattering data used for ω_0 extraction.

Source	f (EHz)	f' (EHz)	θ (°)	Experiment
Cs-137	160.07	69.28 ± 0.5	90 ± 5	[71]
Cs-137	160.07	55.73 ± 0.4	120 ± 5	[71]
Tl-208	632.06	55.13	180	[72]

There will be superior data out there but the conventional Compton experiment is difficult to calibrate, so the variances are quite high. As mentioned, the aim is merely demonstration, so the average value is simply taken by utilizing equation (236) and computing its standard deviation:

$$\bar{f}_0 = \frac{122.1 + 128.2 + 120.8}{3} = 123.7 \pm 3.9 \text{ EHz} \quad (237)$$

Coincidentally, $\bar{f}_0$ lies near the theoretical value derived from CODATA 2022:⁶³

Table 2: CODATA 2022 Recommended Values and Calculated f_0

Constant	Value	Unit
c	299 792 458	m s^{-1}
$\hbar$	$1.054 571 817 \dots \times 10^{-34}$	J s
m_e	$9.109 383 7139(28) \times 10^{-31}$	kg
f_0	123.559	EHz

Of course, there are far superior ways of arriving at a value of ω_0 but here it comes from experimental measurement. It is also readily accessible to university students and can easily be incorporated into undergraduate pedagogy. At present, students measure the Compton wavelength or the electron's rest energy E_e depending on their lab teaching material. That requires χ to serve as a conversion factor. In QED, λ_C is taken to be a mere energy or mass scale with no further explanation offered for its value. There is no explanation for why the electron should behave as something that should have a wavelength—thus, necessarily, a frequency also. From the soliton view, a conserved rest frequency is not simply a curiosity but how the topological defect stays whole. Compton's experiment can determine ω_0 , which is independent of other measured constants as equation (232) deals purely with functions and overall phase differences. It is a wave phenomenon mimicking a kinematic one, not the reverse.

⁶²Please consider that the Tl-208 source is part of the Thorium chain and not the direct source.

⁶³Of course, another choice of selections would be off by 5-10 EHz which still is acceptable taking the variances and systematic errors into account.

11.7.8 Further Commentary on Nonlocality and History

When the electron encounters a field of broadband content in the Fourier space, as an extended body in three dimensions, a purely nonlinear restoring mechanism would not produce the effect seen in Compton scattering. The reasons are twofold: first, a highly nonlinear binding force may produce a polychromatic spectrum rather than the spectrum detected. That should restrict the analysis to bilinearity, even with frequencies beyond the electron's rest frequency. However, bilinearity alone is not sufficient to keep the soliton together! Just as observed in the flexural beam equation, what was necessary was a nonlocal cohesive interaction⁶⁴. The second reason is that there are not enough degrees of freedom in a hypothetical local nonlinear force to fully track the envelope of an incoming wavefront while keeping the soliton intact and radiating a monochromatic wavepacket. According to both experiment and theory, the electron becomes increasingly transparent at higher frequencies, which is not what one would expect if the interaction depended directly on field intensity.

The stability question is not merely qualitative: General instability theorems for nonlinear wave equations, such as Derrick's theorem or that of Karageorgis and Strauss [73], establish that soliton solutions are nonlinearly unstable under certain convexity conditions on the nonlinearity. That being said, these calculations assume the spatial operator is a second-order elliptic differential operator, relying on local spectral theory (Weyl's theorem, Agmon estimates) and Hardy-type inequalities specific to the Laplacian. Lorentz-invariant nonlocal operators lie completely outside their framework. The introduction of an intrinsic length scale through nonlocality modifies the spectral structure and can prevent the exponential growth or finite-time blow-up that drives instability in the local case.

Now, Compton's scattering equation yields only the frequency shift. Obtaining the correct cross-section would demand derivation of the Klein–Nishina formula. This lies beyond the paper's remit, given that electrons have not received a proper spin treatment and so the Dirac equation could not have been derived. Nevertheless, the Compton scattering equation was successfully derived, and the derivation arrives back at the very point where this whole journey began: the electron's rest frame frequency. In a strong sense, understanding the spinless relativistic case builds exactly the correct foundation to treat spin with the same care and hopefully without postulation.

It is somewhat ironic that the same experiment that altered the course of physics history can be viewed as one to motivate a shift into the opposite direction—the Archimedean pivot that could derive all of spinless quantum mechanics up to 1927. What has been presented so far is certainly what Louis de Broglie envisioned when he first encountered the experiment. It is quite surprising that he did not take it all the way; he certainly had all the correct ingredients in his thesis. The matter-wave and soliton are essentially similar ideas, and de Broglie also realized the importance of taking Lorentz boosts into account. Simultaneously, Compton himself was working on extended structures of the electron but abandoned them when he observed what he believed to be kinematic collisions. Would he have been so quick to do so if the concept of the photon was not being popularized in his time? That history cannot be rewritten, but perhaps one can still learn from it.

Even if one rejects the soliton ontology outright, one must still contend with the rest frame frequency ω_0 —a quantity directly measurable through Compton scattering, independent of any theoretical commitment. The measurement needs no $\hbar$, no mass, no

⁶⁴Possibly non-causal at least at the length scale of the electron.

quantum postulates: only the speed of light and a wavelength. From $c = \lambda_C \cdot f_0$, one obtains

$$\omega_0 = \frac{2\pi c}{\lambda_C} \quad (238)$$

a purely wave-mechanical relationship. The Compton wavelength λ_C is measured directly from the scattering shift; the rest follows from the definition of frequency. This single empirical quantity, combined with Lorentz invariance, generates everything derived after Section 7: the Planck and de Broglie relations, the dispersion relation, and ultimately the Klein–Gordon and Schrödinger equations. The soliton provides the *why*; the frequency alone provides the *what*. One may reject the former while accepting the latter, obtaining quantum mechanics without a physical picture—unsatisfying, perhaps, but mathematically complete.

And even if one takes the extreme position of rejecting ω_0 as physically meaningful, insisting it is merely a convenient parameterization with no deeper significance, one is still compelled by parsimony to *postulate* it. For a single postulate—“particles possess a rest frame frequency”—collapses the entire edifice of quantum mechanical postulates into corollaries: the Schrödinger equation, the Planck relation, the de Broglie relation, the Born rule. Seven postulates reduced to one is not interpretation. It is mathematics. The soliton ontology may be debated; the parsimony cannot.

This completes the work.

□

12 Conclusion

To properly conclude this long journey, all that was rediscovered, re-derived and explained along the way must be recounted—most importantly, *how* the results were derived and what was assumed. The paper started with a short but detailed background on the experiments and tensions that partitioned physics into “classical” and “quantum” sectors. With that context in mind, particle ontology was taken seriously. In a sense, the core assumption of the paper is that reality is **real** and so requires an ontological picture—the so-called realist position. This is not taken as a postulate, though understandably some may do so. If reality is real, infinite self-fields cannot exist, nor can contradictions like rigid bodies under Lorentz invariance. The two are mutually entailing—yet both were impossible in reality. What was novel was the process of elimination that cornered the possibilities down to one: a field excitation or topological defect within the field, that is, a soliton.

From there, the classes of solitons available in the literature were investigated. Hopfions suggested a path forward that could account for spin, but the most important classification was whether the electron was fundamental or a breather type. It was concluded that the electron must be fundamental for stability reasons both theoretical and observed. Here charge and energy conservation was necessarily assumed, which is standard and adequately observed⁶⁵. This gave a rest frequency ω_0 , a result arrived at through deduction rather than postulation. By stating that the soliton must survive all field encounters other than encountering its anti-soliton, the standard charge conservation assumption was taken.

Up to that point, the theoretical chain was:

- Every inertial frame must obey Lorentz invariance.
- Reality is real $\Rightarrow$ Point particles and rigid bodies cannot exist $\Rightarrow$ **The only remaining options are persistent and durable field excitations: solitons.**
- Charge and energy are conserved $\Rightarrow$ The soliton must be fundamental with a rest frequency ω_0 .

Though it may not have seemed like it, at the point a fundamental rest frequency and charge conservation were determined, everything that followed was inevitable. The combination of Lorentz invariance, charge and energy conservation produces Maxwell’s equations directly as Poincaré had proven long ago. The rest frame frequency and energy conservation produced Planck and de Broglie’s relations. These relations then defined a dispersion relation for the electron. The extended nature of the soliton required representation as a charge density and phase; combining both manifolds produced a wavefunction ψ . The source-free condition produced a wave equation. The soliton was a field excitation stable within the field. The nonlinear unified field interaction therefore produced the Klein–Gordon equation through a compatibility criterion that eliminated terms in the Lagrangian that over-determined the coupled system. This also produced the Schrödinger equation, solely from the principles itemized earlier.

Nowhere had any quantum assumption or quantization been imported—except in one obvious sense: charge is quantized, which is why electrons only come in one variety. Just as a vortex’s charge and vorticity are conserved, so are electrons, but in addition the

⁶⁵Outside of somewhat poorly modeled ‘weak interactions’.

electron comes in only one variety⁶⁶. As mentioned earlier, the quantum vortices that are solitons showing up in superfluids mimic that exact situation and are similarly quantized. Nevertheless, charge quantization is not in any sense a “quantum” assumption; Hermann von Helmholtz analyzed Faraday’s experiments and concluded that charge was quantized before the electron was even discovered.

The method allowed analysis of all foundational experiments, from blackbody radiation all the way up to Compton scattering, but what was especially dramatic was how the London equation was obtained through the modified Maxwell equation derived alongside the Klein–Gordon equation. No other derivation in the literature has been found that derives these equations from essentially first principles.

Compton’s scattering experiment provided the definitive test, yielding a rest-frame electron frequency alongside the anticipated incident-scattered relationship. This was derived without ever even mentioning the word “photon” or utilizing kinematic arguments, based on an idea Schrödinger proposed in 1927.

All the way back in the beginning of the paper, a promise was made to prove those who claim the Schrödinger equation can only be *guessed* rather than derived directly wrong. That has been done, but the work went much further and necessarily so. The result was a low-energy low-order approximation of an unknown and necessarily Lorentz nonlocal unified field. On that note, the remaining questions follow.

Spin has not been accounted for at all, as a way to control scope. If accounting for a single phase frequency required this much care, one can appreciate the task ahead for those extending this framework to spin and SU(2). Scope control was a must; otherwise this paper would have never seen the light of day. Furthermore, the question that should be haunting the mind of the reader is what form the nonlinear nonlocal Lorentz-invariant restorative force takes. Is there sufficient representation of field interactions to describe this? If so, would the electron be an eigenstate of this equation? How about the remaining “elementary particles”? Would this also cover Hadrons? These are not new questions, but hopefully this paper has made them accessible to a greater number of people.

Now one objection might be that a soliton solution was never actually produced. Fair, but unreasonable. It has been deduced that the solution must exist given the core assumption of realism; all that has been derived came from that existence deduction alone. When Newton deduced $F = ma$ based on Hooke’s law, no one demanded he immediately explain the origin of mass. Has anyone produced the “particle-wave”? No.

Another objection could be the construction of the wave equation in Madelung form out of the phase and density manifolds. Also unreasonable, given that the most natural possible mapping onto a fundamental normed division algebra (complex numbers) has been taken. Any other representation still generates equivalent results but without the convenience of normed division, making representation mathematically cumbersome. One can derive everything that follows with other mathematical objects, but they would not be particularly conducive to calculations. Another possible objection could be halting at dimension 4 for the Lagrangian, but this was justified by analyzing higher dimensionality terms, with dimension 6 being the minimum next order, which gives terms suppressed by powers of energy/mass relative to the leading order Maxwellian term. Yet another objection could be towards the described but not prescribed nonlocality, but that is equivalent to demanding a soliton solution. The analysis does not depend on either; they are deduced to exist as a necessity but do not participate directly in the calculations.

Finally, the strongest objection would be that no new results have been derived, just

⁶⁶Unless one considers the entire lepton family.

a restructuring of how old ones are reached—which is precisely correct. The goal was to overthrow the prevailing pedagogy, the Copenhagen interpretation (or any interpretation as a necessity, as requiring one contradicts realism), and poor formulations in general. Accessibility and historical context were required, even as the technical difficulty escalated. The necessity to postulate many relations, equations and other formulations has been completely removed, including the Planck energy-frequency relation, the Schrödinger equation, the Born rule, and so on. These are all now derived through construction or lowest order derivation.

Towards that end, the realist view has, it is believed, been successfully restored up until 1928⁶⁷ when Dirac successfully derived an equation that accounted for spin. Even then, Dirac’s equation could not anticipate the discovery of Willis Lamb that showed an energy shift due to the electron passing through the nucleus in its $l = 0$ sub-shells. The development since then was rich, and no claim is made to get further than that point. Yet, the possible foundations have been changed significantly. There is nothing in what followed that contradicts the core assumptions, mostly taken for granted. So parsimony, realism, and maximal flexibility for the challenges to come have all been achieved simultaneously.

The reader is hopefully now motivated to collaborate and confront these great challenges with a realist mindset.

This completes the paper. □

⁶⁷Again, ignoring spin, and the Stern-Gerlach experiment.

Appendix A: On the Non-Existence of Photons as Ontic Entities

No experiment has demonstrated the existence of photons as ontic entities propagating through space. What experiments show is that *matter* absorbs and emits electromagnetic energy in discrete packets from nonlinearity.

The “anti-photon” Lamb’s paper and Scully [15] showed that the photoelectric effect admits a semiclassical explanation: when a classical electromagnetic wave encounters matter with discrete energy levels, discrete absorption occurs because the material system can only accept energy matching the allowed transitions of the system. The same reasoning covers the Compton effect, stimulated emission, and laser physics [74]. The “photon” here is not an entity but a means of accounting for rapid nonlinear events involving solitons.

Antibunching of photons [75] is often cited as evidence that semiclassical theory cannot accommodate it. Classical stochastic fields satisfy $g^{(2)}(0) \geq 1$ by the Cauchy-Schwarz inequality: antibunched light violates this bound. Yet, such derivation assumes detectors passively measure time-averaged intensity. Physical detectors remain themselves systems with discrete stable modes. Accounting for this, antibunching emerges from the source dynamics: a single atom cannot emit two excitations simultaneously because the atom must be re-excited after emitting one. The temporal gap reflects relaxation of the *source*, not granularity of the *field* [76].

Khrennikov and colleagues formalized that in prequantum classical statistical field theory (PCSFT) [77, 78], where quantum systems represent classical random fields and all quantum correlations—including entanglement—reduce to classical field correlations. PCSFT reproduces both bunching and antibunching within classical signal theory. No claim is made that PCSFT is a valid framework, but the framework indicates the field-quantizing assertion is sufficient but unnecessary to explain the ‘antibunching’ observation. The framework remains incomplete, lacking the process of elimination that was done for particles.

To that end, the Cauchy-Schwarz inequality bounds correlations for positive-definite probability distributions. Yet classical waves interfere, and interference can produce correlation functions violating these bounds—not from quantization of fields, but because superposition of amplitude differs from superposition of intensity. The “nonclassicality” of antibunching is nonclassicality relative to naive models ignoring wave coherence, not relative to Maxwell’s equations or any modifications of the equations.

In every instance, the “photon” is an epistemic shorthand for energy transferred during material transitions, not an ontic entity. Exactly like a “phonon”. The photon postulate struggles to explain diffraction and interference. Enter ‘wave-particle duality’, a convenient toggle based on the needs of the physicist. That fails the criterion of completeness. A theory positing both wave and particle aspects, or invoking complementarity to avoid choosing, demands interpretation and fails the ontological conditions. The electromagnetic field is a real field from an underlying process that remains undiscovered. To the best of current understanding and experimentation, the field constitutes a continuous entity governed by Lorentz-invariant equations, with nonlinearity at high energies. The field ontology is interpretation-free: what exists is the field: what is observed are the effects of the field on excitations of the field.

Appendix B: On Bell’s Theorem and Nonlocality

Bell’s theorem is frequently cited against non-Copenhagen interpretations⁶⁸, though often without full attention to what the theorem in fact proves. Bell’s theorem proves that there can be no local ‘hidden variable’ theories that reproduce measurable correlations and entanglements produced by the Hilbert space formalism. The framework discussed is explicitly **nonlocal**, the matter here *is* the wave, and the wavefunction is the distributed representation of the wave. There are no hidden variables beneath the field ontology for Bell’s theorem to constrain.

Bell’s theorem does not refute nonlocal realistic theories but *demands them* instead. The correlations observed in EPR-type experiments demand either nonlocality or the abandonment of realism altogether. Bell was unambiguous on this point: Far from viewing Bell’s theorem as refutation of realist interpretations, Bell was a lifelong advocate of the de Broglie-Bohm pilot wave theory, which is an explicitly nonlocal hidden variable theory that Bell’s inequality fails to constrain. Bohm’s 1952 papers first revealed to Bell the fatal flaw in von Neumann’s earlier “proof” that hidden variable theories were impossible [79, 80]. For decades before Bell’s theorem, von Neumann’s invalid no-go theorem stifled development of alternative frameworks. As Bell later wrote: “In 1952 I saw the impossible done” [81]. Bell regarded Bohmian mechanics as a coherent theory, one that demonstrated by explicit construction that quantum phenomena could be understood in terms of an objective reality evolving deterministically—so long as one accepted nonlocality. The only criticism of Bohmian mechanics is the idea of a pilot wave as a separate entity from the field itself which must ultimately violate Lorentz invariance.

Bell’s theorem was never intended to establish that reality is observer-dependent or that hidden variables are impossible. The theorem was intended to show that *locality* isn’t compatible with the observed correlations. The theorem presents a choice: abandon locality, or abandon realism. Bell chose to abandon locality, and so does this framework. Those who cite Bell’s theorem against realist interpretations have, ironically, drawn precisely the opposite conclusion from the one Bell himself drew.

For explicitness: Bell’s theorem is not relevant here; the framework does not deal with local ‘hidden variables’ but a continuous spacetime-filling ontic field. Similar to QED, but without the quantization.

⁶⁸Though do recall how the theory presented here needs no interpretation at all.

Appendix C: On the Origin of χ and α

“All good theoretical physicists put this number $[\alpha]$ up on their wall and worry about it.”
— Paul Dirac

The greatest mystery left is the origin of χ . The constant can be measured to degrees beyond any useful applications. The constant’s relationship to frequency, wavelength, momentum, and energy constitutes all that is known. Yet, ask why the constant takes the particular value that it does, and no one can yield an answer.

An ontological account must only specify what something is. Asking why (in a teleological sense) constitutes another kind of inquiry—one demanding another encompassing deeper ontological framework, that explains the medium itself together with all the excitations of the medium (particles). The soliton mechanics developed here serves as a replacement for quantum mechanics (really, a restructuring that removes all postulates), but not for particle physics (the Standard Model and QED): the mechanics renders no claims about subatomic interactions and cannot classify or predict all the masses, decays, structures, group isomorphisms and symmetries observed over the last century of experimentation.

Yet the parameterization of QED, the Standard Model and the necessary renormalization threshold cutoff decisions have turned particle physics into intense curve fitting lacking epistemology. So the real and final answer is **that there isn’t really any theory out there that explains the origin of χ .**

Tests of scattering remain of no help. The best tests have shown no structure beyond the Compton wavelength at currently achievable energies, though the Schwinger limit remains experimentally unreachable. Simply put, probing the electron itself remains impossible—even within chemistry the calculations involving the charge density of bonds between atoms must rely on calibrated modeling and crystallography. Even so, a rich ‘self-interacting’ field exists in QED and other models, which can be represented in an ontological manner, namely a field configuration along with the appropriate nonlocal nonlinearity that generates the configuration.

At minimum, what is and what is not a valid field configuration can be distinguished. Does the configuration produce a pure electric field beyond some point? Does the configuration reproduce the magnetic moment with a gyromagnetic ratio g that matches the best measurements? Does the model, along with the nonlinear field in which the model resides, reproduce all other detected interactions? Can the model be represented by $SU(2)$? Such are all standard questions, but for a framework to be truly ontological at this level, the framework must do three additional things:

- Explain the origin of mass in the medium.
- Explain why charge only comes in two observable values (+ and -), regardless of the masses of each particle, and why charge does not participate in gravitation.
- Explain the origin of χ , or to be precise, α .

In this section, dimensional analysis is used to create a criterion of plausibility for any combination of electron models and the nonlinear media that would host the models. This first step is expression of χ in terms of a unitless value: the fine structure constant α .

Timelike and Spacelike Self-Energies

Consider the charged electric field of the electron; what is the self-energy of the field? The energy density within that field can be extracted from the Wheeler-Feynman action (80) as purely timelike current given a 'rigid particle' model: $J^\mu(x) = (c\rho(\mathbf{x}), \mathbf{0})$. The current density depends on space only:

$$J_\mu J'^\mu = c^2 \rho(\mathbf{x}) \rho(\mathbf{x}') \quad (239)$$

The current is time-independent, so taking $S_f = UT$ where U is the field energy and T is the total period of integration. All these slices of the integral that have a period of T without loss of generalisation are considered since the value U is constant in time. After a number of reductions, the integral becomes the familiar field energy formula:

$$U_{\text{em}} = \frac{1}{2} \varepsilon_0 \int E^2 dx^3 = \frac{1}{2} \varepsilon_0 \int_a^\infty E^2 r^2 dr \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi \quad (240)$$

$$= \frac{1}{2} \varepsilon_0 \int_a^\infty \left(\frac{e}{4\pi\epsilon_0 r^2} \right)^2 r^2 dr (2)(2\pi) = \frac{e^2}{8\pi\epsilon_0 a} \quad (241)$$

But as Poincaré pointed out, this electron would explode, so the electron needs an equal "stress" around it to hold together, so doubling this energy is needed:

$$E = mc^2 = U_{\text{em}} + U_{\text{stress}}, \quad U_{\text{stress}} = U_{\text{em}} \quad (242)$$

Inverting the relationship:

$$r_e = a = \frac{e^2}{4\pi\epsilon_0 mc^2} \quad (243)$$

A classical result incorporating Poincaré stresses and the 4/3 problem. That construction lacks ontological grounding, since rigid bodies containing 'charge' have already been excised through elimination. Additionally, some unknown mechanism would be needed to hold everything together. Yet dimensional factors of potential **concentration of energy** emerge. Previously mentioned: electrons possess an associated Compton radius (derived from Compton scattering):

$$\bar{\lambda}_C = \frac{\hbar}{mc} \quad (244)$$

Compared to classical radii, Compton radii prove much larger. Compton wavelengths further present thresholds for unattenuated Compton scattering: beyond these, cross-sections drop following $1/\omega$. No further threshold exists past that point, nor any particular one at classical radius r_e . Tantalizing relationships connect these two values. Taking the ratio of the values yields the fine structure constant, quoted to best known values from CODATA 2022:

$$\boxed{\alpha = \frac{r_e}{\bar{\lambda}_C}} = \frac{e^2}{4\pi\epsilon_0 \hbar c} = 0.0072973525643(11) \quad (245)$$

This value is considered a constant in most domains, but in high-energy physics, variability is assumed. Such variation reflects a modeling choice and it is more accurate to consider the value a number expressing a balance of the energy of the electron and

the geometry of the electron enforced by a nonlinear and nonlocal force that keeps the electron stable.

In other words, this value connects the energy domain to the wavelength domain, exactly like χ but has no unit. The value also has an intriguing value of $\approx 1/137$, which makes the value seem like an aspect ratio or a ratio between geometries. Now remember that r_e is purely numerical and no experiment has measured anything resembling structure at that wavelength or any value after the wavelength. One can argue that the same issue applies to the Compton radius—only the Compton wavelength has been truly measured. Calling the wavelength a radius implicitly assumes a geometry.

Can proposed geometries at least be assessed through dimensional analysis to peer further? Only a little bit.

Dimensional Analysis and Ring Vortex Analogy

Recall that a soliton of an electron is not a rigid body but a field excitation with a field configuration. What appears as the effect of charge is the far-field of this object. Much like water surface vortex (whirlpools) create torsion at a distance and move other vortices, such charge is a property of the core of the soliton. In water this core is a singularity in the density of water (which hits zero) around the axis line of the core. Nonlinearity in the medium of water, as expressed by the Navier-Stokes equation, yields a self-sustaining, robust structure.

Now the fun part. Can a vortex serve as an analogy for an electron, yielding a predicted mass? Indeed... though ultimately any estimate involves guessing at charge by assuming a geometry, while ignoring generation of the Coulomb field, spin, and so on. A straight line? A ring? A hopfion-style core swirling in some hard-to-grasp manner? The problem remains under-specified. Still, the analogy deserves pushing as far as the analogy can go.

A thin ring vortex in a medium of density ρ and a vortex charge⁶⁹ Γ , outer radius R , core size a and core characteristic number δ , carries an energy of:

$$E_{\text{vortex}} = \frac{1}{2}\rho\Gamma^2R \left[\ln \left(\frac{8R}{a} \right) - \delta \right] \quad (246)$$

The formula is from Lord Kelvin, derived for a solid core with characteristic $\delta = \frac{7}{4}$. Such a formula has wide applicability, including in Bose-Einstein condensate superfluids where “quantized” line-like vortices are encountered. Keep this result in mind as the electromagnetic analogy is built.

Towards that end, returning to the energy-mass relation which can be written as follows:

$$E = mc^2 = (\sqrt{m}c) \cdot (\sqrt{m}c) \quad (247)$$

If a charge could be extracted from $\sqrt{m}$ the result would resemble the energy stored inside of an inductor:

$$E = \frac{1}{2}LI^2 \quad (248)$$

⁶⁹Not an electric charge but a circulation.

In this framework the inductance is a purely geometric quantity. The current behaves like a closed line charge loop. So far, nothing has been gained over a point-like particle—the singularity has merely spread across another dimension. Now assume this current is a superconducting electromagnetic solution with perfect phase coherency, eliminating the need for considering time. The Wheeler–Feynman action can then be applied again with a purely spacelike current $J^\mu = (0, \mathbf{J})$ and an energy calculated via the resultant Biot-Savart law.

$$U = \frac{\mu_0}{8\pi} \iint \mathbf{J}(\mathbf{r}) \cdot \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r d^3r' \quad (249)$$

Separating out the current along a path yields the Neumann inductance integral. For a thin-wire single-turn loop self-inductance of outer radius R and core size a , the asymptotic inductance energy is:

$$E = \frac{1}{2} \mu_0 I^2 R \left[\ln \left(\frac{8R}{a} \right) - \delta \right] \quad (250)$$

So a coil inductor exhibits a similar energy to a vortex but the current has a different dimensional sense. Temptation arises, then, for seeing if the predicted mass for a particular geometry together with core distribution δ can be extracted. Giving in to temptation, just for illustration:

$$m_{\text{pred}} = \frac{1}{2} \mu_0 \left(\frac{I}{c} \right)^2 R \left[\ln \left(\frac{8R}{a} \right) - \delta \right] \quad (251)$$

Now the vortex charge (circulation, units of m^2/s) has a related entity: the charge density I/c ; the medium density ρ becomes the permeability μ_0 . Unfortunately, the expression is under-specified. How does one determine a and R , or smear a charge e across a path? Yet with some effort the expression can be made usable. First establishing a general form for the current scaled by the total size to get the correct units:

$$I = \gamma \frac{ec}{R} \quad (252)$$

Here the simplest case of a ring electron with the charge smeared across a circle would be characterized by $\gamma = 1/(2\pi)$. Then with scaling factors on the Compton radius and the classical radius to allow for dimensional choice:

$$R = \beta \bar{\lambda}_C, \quad a = \kappa \alpha R \quad (253)$$

to rewrite the formula this way:

$$\frac{m_{\text{pred}}}{m} = 2\pi\alpha \frac{\gamma^2}{\beta} \left[\ln \left(\frac{8}{\kappa\alpha} \right) - \delta \right] \quad (254)$$

Not all of these parameters are independent: the parameters do have to produce physical sense, but the parameters can characterise toy models that smear a charge around a toroid. If the combination of β , γ , κ and δ can recover α and the correct predicted mass, then the combination passes this test. If it cannot, one would need to find a fault in such analysis. For example, demand would arise that velocity of phase would be used instead, but then reliance on Maxwell's equations really couldn't occur (an improved understanding of the medium would be needed, perhaps a relativistic fluid concept developed in future work).

No one's model will be singled out, but trying this formula out for the typical setup is worthwhile. A popular choice is $\beta = 1$, $\kappa = 1$. The sensitivity of α to δ proves minute but the NLSE $\delta = 1.615$ from Bose-Einstein condensates [82] may be substituted. What should be used for γ ? An almost perfect fit is simply 2. Substituting the fine structure constant yields a prediction ratio of:

$$\frac{m_{\text{pred}}}{m} = 98.75\% \quad (255)$$

Which is quite impressive—indeed, cause for premature celebration... until realizing that the γ factor is impossible—a ring electron should have a γ equal to $1/(2\pi)$ as explained before. A value of 2 would only make sense if the charge were half the radius of the Compton wavelength which cannot fit a toroidal geometry.⁷⁰

Instead of predicting mass, suppose the mass needs to be an absolute fit: then what would the fine structure constant need to be?

$$\frac{1}{\alpha} = -\frac{2\pi\gamma^2}{\beta} W_{-1} \left(-\frac{\beta\kappa e^\delta}{16\pi\gamma^2} \right) \quad (256)$$

with W_{-1} being the sub-principal branch of the Lambert W function and the parameter domain of the function is $[-1/e, 0)$.

By differentiating this expression the result proves quite insensitive in the typical parameter ranges to changes in δ which characterizes the inner structure and the medium that carries the structure. The restriction has killed off most naive toroidal “charge lines moving at speed of light c ” geometries that have been thrown at the restriction. **The toy ring electron theory must be rejected** as physically impossible on geometric grounds.

Where does that leave things, in terms of understanding α ? Perhaps not further away, but with greater appreciation for the challenge the constant imposes. Then there is the gyromagnetic ratio, the SU(2) group action, the single frequency of phase restriction, the scattering profiles and the Compton scattering relations. To get any further, a much improved understanding of the medium and how the medium operates at the finest levels is needed. The favored candidate is a standing wave that self-intersects as a Hopf fibration but here there is no charge and really no physics either. At minimum the candidate *must* exist, unlike a rigid body or a point particle.

A nonlinear, likely nonlocal, description of the medium along with a topological account of all particles is needed. Only then can the singular question be answered: What does α truly mean?

If the answer seems too far away or impossible, always remember that physicists dreamed of knowing what is known today just two hundred years ago. Yet caution is advised; only the utmost honesty in evaluating models and respect for experimental results will unlock the truth. Perhaps the reader will have the boldness, the excellent ability to collaborate and integrity to crack this mystery?

On the Reported Universality of $\chi = \hbar$

So far, the χ relationship has only been covered for electrons, but the framework and the experimental data to follow will show an exact match with $\hbar$, which is not exactly

⁷⁰The choice of 2 was a way to represent a double cover but this was an inversion of the magnitude. The silver lining was this full analysis!

surprising since the exact same dynamics used in these experiments were derived.

Almost all experiments ever conducted by humans that verified the Planck relation used electrons in the experimental apparatus (for example in Bragg deflectors) or measured wave packets emitted by electrons. The most accurate test uses angular soliton flux to determine $\hbar$ with dramatic precision. There is one exception however: Muonic hydrogen, where the electron orbiting the proton is replaced by a heavier lepton called a muon. The transitions between the orbitals are measured which could in theory yield a different fine structure and reduced mass compared to regular hydrogen. These incredible experiments are now famous for numerous anomalies, the most familiar of which is the radius of the proton appearing to be different from earlier measurements. Yet it is quite likely that the ratio is almost the same if not identical, given how small these anomalies remain (except for the implied mass of the proton).

It would not be surprising if people discover that $\hbar$ is very specific to electrons at some point in the distant future, or at least to leptons, and has to be perturbed for deepening understanding of hadrons and the like.

Epilogue: On the Meaningless Classification of ‘Classical’ and ‘Quantum’

After publishing this work, I will find it difficult to speak of any separation between classical and quantum mechanics—so much here derives from classical physics alone. All the material used was developed during the 1700s and 1800s; nobody can claim anything was smuggled in from quantum mechanics. Though once well-intentioned, “quantization” has become a meaningless term, simultaneously overloaded. Does the term mean discretization of energy packets? The Planck relation? Does the term mean photon existence (which was refuted in an earlier section, aligning with Willis Lamb and others)? Does the term mean arbitrary spatial discretization used for computation? Or does the term really mean restricting resonant modes to discrete ranges? Ever since I was an undergraduate, I have never received a satisfactory answer.

As a student, I was told quantum mechanics was something not to understand but simultaneously to master completely. A seemingly endless series of ansätze and abstractions that looked fairly disconnected from everything learned before. Simultaneously, all earlier material was marked somewhat ‘obsolete’ or ‘classical’, even though quantum mechanics **needs** this material for macroscopic lab descriptions. I could see the correspondence directly when synthesizing the bilinear interaction between both fields. For me, this was heartbreaking. I had absolute love for the field and wanted to devote my life to it.

My goal in writing this work was a total reorganization of the ontological picture that has been missing for one hundred years. Throughout, I stuck with the symbol χ in place of $\hbar$ to make one thing about the reconstruction clear: no quantum machinery was smuggled in. This may seem like a point of vanity, but reflexively, such things are assumed to have surreptitiously slipped into a paper when $\hbar$ appears. Whether this work succeeds in its goal depends on whether people truly want physics to be physical or remain an undecipherable mess for students told to “shut up and calculate”. A real physicist always has a massive picture of what they are working on in their mind. This picture should move, should illuminate their day and inspire them to dig deeper while increasing breadth.

As such, hopefully this work *has* brought inspiration and illuminated your world. Thank you so much for the effort reading my writing must have taken—I am truly grateful from the bottom of my heart.

Yours forever,

Korobochka

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